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Lord LINDSAY, M.P., F.R.S., President, in the Chair.

The Rev. Charles Heaven, M.A., The Orchards, Stone, Aylesbury; and
James Whewall, Esq., Savile House Observatory, Halifax;

were balloted for and duly elected Fellows of the Society.

On the Law of Force to any Point in the Plane of Motion, in order that the Orbit may be always a Conic.

By J. W. L. Glaisher, M.A., F.R.S.

§ 1. In a paper "Sur la possibilité de déduire d'une seule des lois de Kepler le principe de l'attraction," printed in the *Comptes Rendus*, t. 84, pp. 671–674 (April 9, 1877), M. Bertrand remarks that it would be interesting to solve the following problem: "En sachant que les planètes décrivent des sections coniques, et sans rien supposer de plus, trouver l'expression des composantes de la force qui les sollicite, exprimées en fonction des coordonnées de son point d'application," and adds, "Nous connaissons deux solutions: La force peut-être dirigée vers un centre fixe et agir proportionnellement à la distance, ou en raison inverse de son carré. En existe-t-il d'autres?"

The problem was completely solved by M. Darboux (*Comptes Rendus*, t. 84, pp. 760–762 and 936–938, April 16 and 30), and M. Halphen (*Ibid.*, pp. 939–941), who each proved that a body moving in the plane of xy under the action of a central force, equal to either

$$\frac{\mu r}{(ax+by+c)^n} \quad \ldots\ldots\ldots (1)$$

or

$$\frac{\mu r}{(ax^2+bxy+cy^2)^{\frac{5}{2}}} \quad \ldots\ldots\ldots\ldots \quad (2)$$

must describe a conic whatever be the initial conditions of the motion, and that there are no other laws of force for which this is true.

It is also shown that all the conics described under the action of the law (1) have $ax+by+c=0$ as polar of the origin, and those described under the action of (2) have $ax^2+bxy+cy^2=0$ as tangents (real or imaginary).

M. Darboux obtained his results by substituting the value of u given by the general equation of a conic,

$$u = a\cos\theta + b\sin\theta + \sqrt{(\mathrm{A}\cos 2\theta + \mathrm{B}\sin 2\theta + \mathrm{H})},$$

in the differential equation

$$\frac{d^2u}{d\theta^2}+u=\frac{\mathrm{P}}{h^2u^2};$$

while M. Halphen's process, which is very elegant, depends upon the differential equation of the fifth order satisfied by conics.

If the motion is not restricted to take place in the plane of xy, the laws of force are

$$\frac{\mu r}{(ax+by+cz+d)^3}$$

and

$$\frac{\mu r}{(ax^2+a'y^2+a''z^2+2byz+2b'zx+2b''xy)^{\frac{5}{2}}};$$

and, as remarked by M. Halphen, in the former case all the polars of the centre of force with regard to the conics lie in the plane $ax+by+cz+d=0$, and in the latter case all the conics have double contact with the cone

$$ax^2+a'y^2+a''z^2+2byz+2b'zx+2b''xy=0.$$

In what follows I shall, for the sake of simplicity, only consider motion in the plane of xy; it is evident that the extension to the case of motion in three dimensions presents no difficulty.

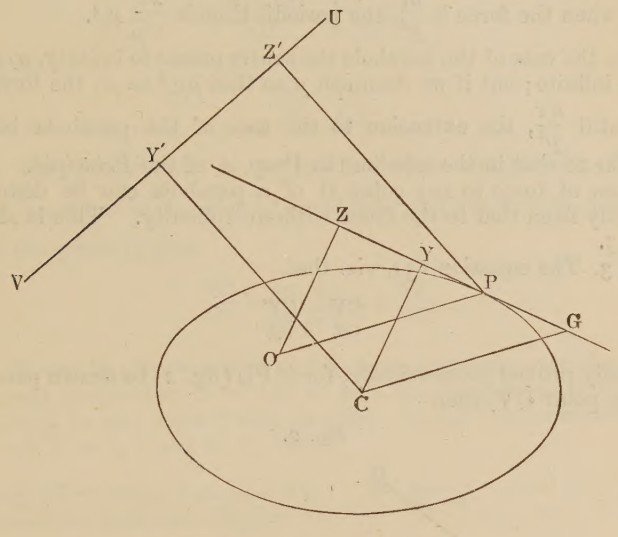
§ 2. Newton, in the scholium to Prop. xvii. of the *Principia*, finds the law of force tending to any point in order that any given conic may be described about it; and I now proceed to consider the expression for the force given by Newton, and to connect it with the results obtained by MM. Darboux and Halphen.

The law of force in the scholium to Prop. xvii. is deduced by Newton from Corollary 3 to Prop. vii., in which it is shown that the force under the action of which a body P revolves in any

orbit about any centre of force C is to the force under the action of which it can revolve in the same orbit in the same periodic time about any other centre of force O as $OP^2 . CP$ to CG^3, CG being drawn parallel to OP and cutting the tangent at P in G.

Now let C be the centre of the conic, so that the law of force is μr; then (fig. 1)

Fig. 1.



$$\frac{\text{force to O}}{\text{force to C}} = \frac{CG^3}{OP^2 . CP},$$

therefore

$$\text{force to O} = \mu \left(\frac{CG}{OP}\right)^3 OP,$$

which is the law of force given in the scholium to Prop. xvii.

If CY and OZ are the perpendiculars let fall upon the tangent at P, we see that

$$\text{force to O} = \mu \left(\frac{CY}{OZ}\right)^3 OP.$$

Now if UV is the polar of O, and CY′, PZ′ are perpendiculars let fall upon it,

$$\frac{CY}{OZ} = \frac{CY'}{PZ'} \quad \ldots\ldots\ldots\ldots \quad (3)$$

so that

$$\text{force to O} = \mu \left(\frac{CY'}{PZ'}\right)^3 OP$$

$$= \mu \left(\frac{p_o}{p}\right)^3 r,$$

p and p_o denoting the perpendiculars from P and the centre upon the polar of O.

Thus any conic may be described under the action of a force $\frac{\mu r}{p^3}$ tending to O; and since the periodic time is the same for the force μr to C and $\mu \left(\frac{p_o}{p}\right)^3 r$ to O, it is equal to $\frac{2\pi}{\sqrt{\mu}}$; and therefore when the force is $\frac{\mu r}{p^3}$, the periodic time is $\frac{2\pi}{\sqrt{\mu}} p_o^{\frac{3}{2}}$.

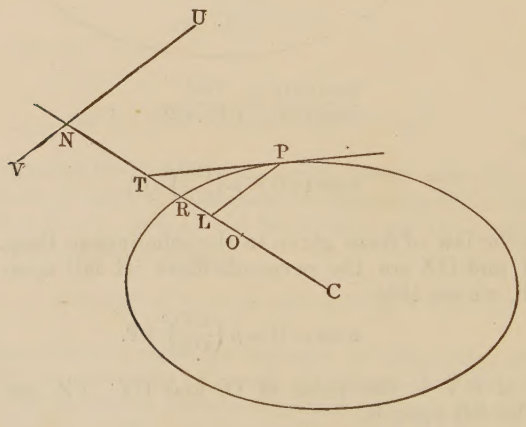
In the case of the parabola the centre passes to infinity, so that p_o is infinite; but if we diminish μ so that $\mu p_o^3 = \mu'$, the force to O is still $\frac{\mu' r}{p^3}$, the extension to the case of the parabola being similar to that in the scholium to Prop. x. of the *Principia*. But the law of force to any point O of a parabola can be deduced directly from that to the focus without difficulty. This is done in § 5.

§ 3. The equation (3), viz. that

$$\frac{CY}{OZ} = \frac{CY'}{PZ'},$$

is easily proved geometrically, for if PL (fig. 2) be drawn parallel to the polar UV, then

Fig. 2.



$$\frac{\text{perpendicular from O on tangent at P}}{\text{perpendicular from C on tangent at P}} = \frac{OT}{CT},$$

$$\frac{\text{perpendicular from P on polar of O}}{\text{perpendicular from C on polar of O}} = \frac{LN}{CN},$$

and

$$\frac{OT}{CT} = \frac{LN}{CN},$$

for

$$CO \cdot CN = CR^2 = CL \cdot CT,$$

giving

$$\frac{CO}{CT} = \frac{CL}{CN};$$

and therefore

$$\frac{CT - CO}{CT} = \frac{CN - CL}{CN},$$

that is

$$\frac{OT}{CT} = \frac{LN}{CN};$$

but the proposition is evident at once analytically, for if $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be the equation of the ellipse, and if h, k be the point O, x, y the point P, then

$$\frac{OZ}{CY} = \frac{hx}{a^2} + \frac{ky}{b^2} - 1 = \frac{PZ'}{CY'}.$$

It may be observed that (3) is only a particular case of a more general proposition in which P and the tangent at P may be replaced by any point P and its polar; for let the coordinates of O and P be h, k and h', k', then

$$\frac{\text{perpendicular from P on polar of O}}{\text{perpendicular from centre on polar of O}} = \frac{hh'}{a^2} + \frac{kk'}{b^2} - 1$$

$$= \frac{\text{perpendicular from O on polar of P}}{\text{perpendicular from centre on polar of P}}.$$

This proposition is proved for circles in Salmon's *Conics*, 5th edition, Art. 101, p. 93.

§ 4. The law of force in the Newtonian form $\mu \left(\frac{CY}{OZ}\right)^3 OP$ leads directly to the analytical expressions for the force in terms of the coordinates of any point; for let the point O be the origin, and suppose that the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is described about O. The tangent at x, y is

$$ax\xi + h(x\eta + y\xi) + by\eta + g(x + \xi) + f(y + \eta) + c = 0;$$

and, x_0, y_0 being the coordinates of the centre,

$$\frac{CY}{OZ} = \frac{axx_0 + h(xy_0 + yx_0) + byy_0 + g(x + x_0) + f(y + y_0) + c}{gx + fy + c},$$

where x_o, y_o are given by the equations

$$ax_o + hy_o + g = 0,$$

$$hx_o + by_o + f = 0.$$

Multiplying these equations respectively by x and y, and subtracting them from the numerator of the expression for $\frac{CY}{OZ}$, we have

$$\frac{CY}{OZ} = \frac{gx_o + fy_o + c}{gx + fy + c},$$

and therefore

$$\text{force to O} = \mu\left(\frac{CY}{OZ}\right)^3 OP = \mu\left(\frac{gx_o + fy_o + c}{gx + fy + c}\right)^3 r$$

$$= \frac{\mu' r}{(gx + fy + c)^3}.$$

§ 5. Consider the case of the parabola separately. The force to the focus S is $\frac{\mu}{SP^2}$, and therefore

$$\text{force to any point O} = \left(\frac{Sg^3}{OP^2 . SP}\right)\frac{\mu}{SP^2} = \mu\left(\frac{Sg}{OP . SP}\right)^3 OP$$

$$= \mu\left(\frac{Sy}{OZ . SP}\right)^3 OP,$$

where Sg is drawn parallel to OP cutting the tangent at P in g, and Sy is the perpendicular upon the tangent at P.

Now $\frac{Sy}{SP} = \sin\theta$, where θ is the inclination of the tangent at P to the axis of the parabola, and therefore

$$\text{force to O} = \mu\left(\frac{\sin\theta}{OZ}\right)^3 OP \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$= \mu\left(\frac{\sin\alpha}{PZ'}\right)^3 OP,$$

where PZ′ is the perpendicular upon the polar of O, and α is the inclination of the polar of O to the axis of the parabola. Thus the force to O $= \frac{\mu' r}{p^3}$ as before.

The theorem

$$\frac{OZ}{\sin\theta} = \frac{PZ'}{\sin\alpha}$$

is true when P and the tangent at P are replaced by any point P and the polar of P; viz. we have

$$\frac{\text{perpendicular from P on polar of O}}{\text{perpendicular from O on polar of P}} = \frac{\sin\alpha}{\sin\beta},$$

where α, β are the inclinations of the polars of O and P to the axis of the parabola.

Enunciated in a geometrical form, this theorem asserts that if O and P be any two points, and if OO′ be drawn parallel to the axis, cutting the polar of P in O′, and PP′ be drawn parallel to the axis, cutting the polar of O in P′, then OO′ = PP′. The proof is very simple; for, $y^2 = 4ax$ being the equation of the parabola and h, k the point O, h', k' the point P, then the equation of the polar of P is $yk' = 2a\,(x+h')$, and therefore

$$OO' = \frac{kk'}{2a} - h - h',$$

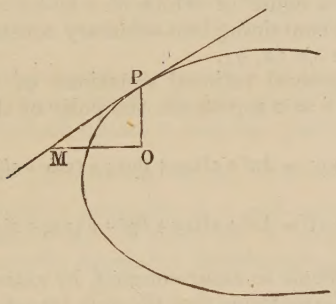
which involves the coordinates of O and P symmetrically.

The equation (4) gives a simple expression for the law of force to any point in a parabola; for if a body be describing a parabola under the action of a force tending to any point O, then we see from (4) that

$$\text{force to O} = \mu \frac{OP}{OM^3},$$

where OM is drawn parallel to the axis cutting the tangent at P in M (fig. 3).

Fig. 3.



§ 6. It follows therefore that under the action of the force

$$\frac{\mu r}{(ax+by+c)^3} \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

any conic having $ax+by+c = 0$ as the polar of O may be described. Taking O as the origin, the general equation of all such conics is

$$(ax+by+c)^2 = Ax^2 + 2Hxy + By^2; \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and since, the line $ax+by+c = 0$ being given, this equation contains two arbitrary constants in addition to an arbitrary quantity λ by which a, b, c must admit of being multiplied in virtue of

the factor $\frac{\mu}{h^2}$ in the differential equation, it follows that this is the most general form of orbit that can be described under the action of the force (1). From (5) we see that the conic can be also described under the action of the force

$$\frac{\mu r}{(Ax^2+2Hxy+By^2)^{\frac{3}{2}}}; \quad \ldots\ldots\ldots \quad (2)$$

and, since $Ax^2+2Hxy+By^2=0$ being two given lines, the equation (5) contains two arbitrary constants besides the quantity λ by which A, B, C may be multiplied, it follows that, under the action of (2), the body will always describe a conic having $Ax^2+2Hxy+By^2=0$ as the (real or imaginary) tangents from the origin.

§ 7. There are no other laws of force for which the orbit is always a conic; for suppose that

$$\frac{\mu r}{\{\phi(x, y)\}^3}$$

expresses such a law, then

$$ax+by+c=\phi(x, y) \quad \ldots\ldots\ldots \quad (6)$$

must represent a conic of which $ax+by+c=0$ is the polar of the origin, and containing two arbitrary constants in addition to the constants in $\lambda\phi(x, y)$.

The most general rational equations of the form (6) for which $ax+by+c=0$ represents the polar of the origin are

$$ax+by+c = Ax^2+2Hxy+By^2+\nu(2ax+2by+c)+\frac{1}{2}c. \quad \ldots \quad (7)$$

$$(ax+by+c)^2 = Ax^2+2Hxy+By^2+\nu(2ax+2by+c) \quad \ldots \quad (8)$$

and it is not difficult to assure oneself, by examining the different cases and transformations, that the only transformation satisfying the requisite conditions is obtained by putting $\nu=0$ in (8), which then becomes identical with (5). M. Darboux gives a proof that (1) and (2) are the only laws, on pp. 936, 937 of his second paper in the *Comptes Rendus*.

From (7) we see that if the force be

$$\frac{\mu r}{(Ax^2+2Hxy+By^2+f)^3}$$

the body will describe a conic having its asymptotes parallel to the lines $Ax^2+2Hxy+By^2=0$ if properly projected.

It will be observed that the problem solved by Newton is, "Given any conic (or curve, the law of force for the description of which about any one point is known), find the law of force to

any point"; while M. Bertrand's problem is, "Find the general law of force, expressed in terms of x and y, such that the orbit is always a conic, whatever the initial conditions may be," and it is remarkable that there should be two distinct laws of force for which this is true.

§ 8. As far as I know, the pair of expressions (1) and (2) were first obtained by MM. Darboux and Halphen; but the law $\frac{\mu r}{p^3}$, which is identical with $\frac{\mu r}{(ax+by+c)^3}$, was discovered by Sir W. R. Hamilton, and is thus enunciated in the *Proceedings* of the Royal Irish Academy, No. 57, vol. iii., p. 308 (November 30, 1846).

"Sir William R. Hamilton stated the following theorems of central forces, which he had proved by his calculus of quaternions, but which, as he remarked, might be also deduced from principles more elementary. 'If a body be attracted to a fixed point, with a force which varies directly as the distance from that point and inversely as the cube of the distance from a fixed plane, the body will describe a conic section, of which the plane intersects the fixed plane in a straight line, which is the polar of the fixed point with respect to the conic section."

The second theorem, which is analogous to the first, relates to motion on a sphere in a spherical conic. The account then proceeds: "The first theorem had been suggested to Sir W. Hamilton by a recently resumed study of a part of Sir Isaac Newton's *Principia*."

A short analytical proof of the converse of this theorem, viz. that "if X be any given conic, O any point in its plane, F a central force at O which causes a material particle to describe X, then F varies as $\frac{r}{p^3}$," was given by Professor Casey in a paper, "On M'Cullagh's property of a self-conjugate triangle and Sir W. Hamilton's law of force for a body describing a conic section" (*Quarterly Journal of Mathematics*, vol. v. (1862), pp. 233–235).

It thus appears that a reply to M. Bertrand's question with regard to the existence of other laws besides μr and $\frac{\mu}{r^2}$ for which the orbit was always a conic had been given by Sir W. R. Hamilton's theorem that the orbit is always a conic when the law of force is $\frac{\mu r}{p^3}$, p being the perpendicular on any fixed plane; but M. Bertrand's general problem seems never to have been considered. It is not unlikely that Sir W. R. Hamilton deduced his law from Newton's Prop. xvii., and it is curious that the general formulæ which are deducible from the translation of Newton's results into analysis should not have been examined.

§ 9. The elegant expression for the periodic time to which Newton's corollary leads, viz. that if a conic be described under

the action of the force $\frac{\mu r}{p^3}$ tending to a point O, then the periodic time is $\frac{2\pi}{\sqrt{\mu}} p_o^{\frac{3}{2}}$, where p_o is the perpendicular from the centre of the conic upon the polar of O, seems not to have been previously noticed. It is interesting to apply it to the cases in which O coincides with the centre and focus. If O coincides with the centre, then the polar is at infinity, so that $\frac{\mu}{p^3} = \frac{\mu}{p_o^3}$ $=$ constant $=\mu'$ suppose: thus the force is $\mu' r$ and the periodic time is $\frac{2\pi}{\sqrt{\mu'}}$.

If O coincides with the focus, the polar is the directrix, so that $p = \frac{r}{e}$: thus the force is $\frac{\mu e^3}{r^2}$ and the periodic time is $\frac{2\pi}{\sqrt{\mu}} \left(\frac{a}{e}\right)^{\frac{3}{2}}$, that is, the force is $\frac{\mu'}{r^2}$ and the periodic time is $\frac{2\pi}{\sqrt{\mu'}} a^{\frac{3}{2}}$.

If p denotes, as in Sir W. R. Hamilton's theorem, the perpendicular upon a given fixed plane, the periodic time will be $\frac{2\pi}{\sqrt{\mu}} p_o^{\frac{3}{2}}$, p_o being the perpendicular from the centre upon the fixed plane; so that the periodic time in all orbits having their centres in a plane parallel to the fixed plane is the same. If the plane of motion be parallel to the fixed plane p is constant and we have the case of motion under the action of a force to the centre.

§ 10. By means of the formula $\frac{2\pi}{\sqrt{\mu}} p_o^{\frac{3}{2}}$ it will be found that if the conic

$$(ax+by+c)^2 = Ax^2+2Hxy+By^2$$

be described about the origin under the action of the force

$$\frac{\mu r}{(ax+by+c)^3}$$

or

$$\frac{\mu r}{(Ax^2+2Hxy+By^2)^{\frac{3}{2}}},$$

the periodic time will be

$$\frac{2\pi}{\sqrt{\mu}} (ax_o+by_o+c)^{\frac{3}{2}},$$

x_o, y_o being the coordinates of the centre,

$$= \frac{2\pi}{\sqrt{\mu}} \left\{ \frac{c\,(AB-H^2)}{AB-H^2-(Ab^2+Ba^2-2Hab)} \right\}^{\frac{3}{2}};$$

and that if the conic

$$ax^2+2hxy+by^2+2gx+2fy+c = 0$$

be described about the origin under the action of the force

$$\frac{\mu r}{(gx+fy+c)^3},$$

or

$$\frac{\mu r}{\{(g^2-ac)\,x^2+2\,(fg-hc)\,xy+(f^2-bc)\,y^2\}^{\frac{3}{2}}},$$

the periodic time will be

$$\frac{2\pi}{\sqrt{\mu}}(gx_0+fy_0+c)^{\frac{3}{2}},$$

$$=\frac{2\pi}{\sqrt{\mu}}\left\{\frac{c\,(ab-h^2)-(af^2+bg^2-2hfg)}{ab-h^2}\right\}^{\frac{3}{2}},$$

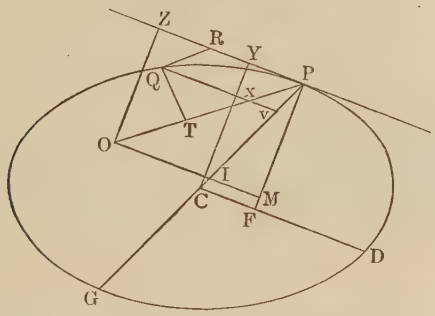
and the same expression will denote the periodic time if this orbit be described about the point p, q under the action of the force

$$\frac{\mu r}{\{apx+h\,(xq+yp)+bqy+g\,(x+p)+f\,(y+q)+c\}^3},$$

r being measured from p, q.

§ 11. The expression $\mu\left(\frac{\mathrm{CY}}{\mathrm{OZ}}\right)^3\mathrm{OP}$ for the force to any point O may be investigated directly in the same manner and with the same facility as the force to the focus is obtained by Newton in Prop. xi.; for if F denote the force to O, then (fig. 4)

Fig. 4.



$$\mathrm{F}=\frac{2h^2}{\mathrm{OP}^2}\cdot\frac{\mathrm{QR}}{\mathrm{QT}^2}.$$

By similar triangles QTx, PMO,

$$\frac{\mathrm{QT}}{\mathrm{Qx}}=\frac{\mathrm{PM}}{\mathrm{OP}},$$

and

$$\frac{\mathrm{Qv}^2}{\mathrm{Pv}\cdot\mathrm{Gv}}=\frac{\mathrm{CD}^2}{\mathrm{CP}^2}. \quad \ldots\ldots\ldots \quad (9)$$

Now

$$\frac{Pv}{QR} = \frac{Pv}{Px} = \frac{PI}{OP} = \frac{CP}{PF}\cdot\frac{PM}{OP};$$

therefore

$$\frac{Qv^2}{QR\,.\,Gv} = \frac{CD^2}{CP\,.\,PF}\cdot\frac{PM}{OP},$$

whence, since ultimately $Qx = Qv$, and by (9),

$$\frac{QT^2}{QR} = 2\,\frac{CD^2}{PF}\left(\frac{PM}{OP}\right)^3.$$

Thus

$$F = \frac{2h^2}{OP^2}\cdot\frac{1}{2}\,\frac{PF}{CD^2}\left(\frac{OP}{PM}\right)^3$$

$$= \frac{h^2}{a^2b^2}\left(\frac{PF}{PM}\right)^3 OP$$

$$= \frac{h^2}{a^2b^2}\left(\frac{CY}{OZ}\right)^3 OP.$$

Newton himself uses Corollary 3 of Prop. vii. in his second demonstration of Prop. xi. to deduce the law of force to the focus from that to the centre.

§ 12. The law of force

$$\frac{\mu r}{(Ax^2 + 2Hxy + By^2)^{\frac{5}{2}}}$$

is connected with the known equations for motion about the focus in an elliptic orbit in a plane inclined to the plane of reference; for if a sphere described with the focus as centre cut the plane of the orbit and the plane of reference in NP and NM, and if PM be a great circle perpendicular to NM, then $PM = s$, and if u denote the reciprocal of the projection of the radius vector upon the plane of reference, it is well known that

$$lu = \sqrt{(1+s^2)} + e\cos(\theta - \varpi) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where l is the semi-latus rectum of the orbit. But if h denote twice the area described in unit of time by the projection of the radius vector,

$$\frac{d^2u}{d\theta^2} + u = \frac{P\cos PM}{h^2u^2}$$

$$= \frac{\mu}{h^2}\cos^3 PM$$

$$= \frac{\mu}{h^2}\,\frac{1}{(1+s^2)^{\frac{3}{2}}},$$

Now $s = k \sin(\theta - \gamma)$, so that the differential equation is

$$\frac{d^2u}{d\theta^2} + u = \frac{\mu}{h^2} \frac{1}{\{1 + k^2 \sin^2(\theta - \gamma)\}^{\frac{3}{2}}},$$

while (10) becomes

$$lu = \sqrt{\{1 + k^2 \sin^2(\theta - \gamma)\}} + e \cos(\theta - \varpi);$$

and therefore, transforming from polar to rectangular coordinates, if the law of force is

$$\frac{\mu'}{\{(1 + k^2 \sin^2\gamma)\, x^2 - 2k^2 \sin\gamma \cos\gamma\, xy + (1 + k^2 \cos^2\gamma)\, y^2\}^{\frac{3}{2}}},$$

the equation of the orbit is

$$(1 + k^2 \sin^2\gamma)\, x^2 - 2k^2 \sin\gamma \cos\gamma\, xy + (1 + k^2 \cos^2\gamma)\, y^2 = (Ax + By + l)^2,$$

agreeing with (5). The ordinary equations of elliptic motion thus indicate the laws (1) and (2); and these laws may be found directly by considering the orthogonal projections of a conic on a plane through the focus.

For, the equation of a conic having the origin for focus is

$$x^2 + y^2 = (Ax + By + C)^2;$$

and if this be projected orthogonally upon a plane through the focus, the axis of x being the line of intersection of the two planes, the equation of the projection is of the form

$$x^2 + m^2y^2 = (Ax + Bmy + C)^2;$$

that is, any conic having the tangents from the origin equally inclined to the axes, and as we may turn the axes of coordinates through an arbitrary angle, we thus obtain any conic.

§ 13. Newton's law, $\mu \frac{CG^3}{OP^2}$, for the force to O may be put in the form

$$\mu \frac{DD'^6}{OP^2 . PP'^3} \quad \ldots\ldots\ldots \quad (11)$$

where PP′ is the chord through O and DD′ is the diameter parallel to the chord PP′; for if PM be drawn parallel to the line joining C to the middle point of PP′, cutting CD in M, then $CG.CM = CD^2$, and therefore $2CG.PP = DD'^2$. The form (11) was given in an examination paper set in St. John's College, Cambridge, in December 1869, and I am indebted to Mr. E. J. Routh for the reference to it.

By a known theorem, if a, b be the semi-axis major and semi-axis minor of an ellipse

$$\mathrm{PP'} = \frac{1}{2a}\,\mathrm{DD'^2}\sqrt{\left(1-\frac{p_1p_2}{b^2}\right)},$$

p_1, p_2 being the perpendiculars upon the chord PP′ from the foci S, H; and therefore, if the body describe an elliptic orbit about O,

$$\text{force to O} = \frac{\mu}{r^2\,(b^2-p_1p_2)^{\frac{3}{2}}},$$

where $\mathrm{OP} = r$ and p_1, p_2 have the same or opposite signs according as S, H are on the same or on opposite sides of PP′.

If the orbit be hyperbolic

$$\text{force to O} = \frac{\mu}{r^2\,(b^2+p_1p_2)^{\frac{3}{2}}},$$

b being the semi-conjugate axis.

Consider the case when the orbit is an ellipse. Since

$$\frac{\mathrm{CG}^3}{\mathrm{OP}^2} = \left(\frac{p_o}{p}\right)^3 r, \text{ by § 2,}$$

and also,

$$= \frac{1}{r^2}\left(\frac{\mathrm{DD'^2}}{2\mathrm{PP'}}\right)^3 = \frac{1}{r^2}\frac{(ab)^3}{(b^2-p_1p_2)^{\frac{3}{2}}},$$

it follows that

$$\frac{p_o}{p} = \frac{1}{r}\frac{ab}{\sqrt{(b^2-p_1p_2)}}.$$

Now, when the force $= \dfrac{\mu r}{p^3}$, the periodic time $= \dfrac{2\pi}{\sqrt{\mu}}\,p_o^{\frac{3}{2}}$, and therefore when the force

$$= \mu\frac{a^3b^3}{{p_o}^3}\frac{1}{r^2}\frac{1}{(b^2-p_1p_2)^{\frac{3}{2}}},$$

the periodic time $= \dfrac{2\pi}{\sqrt{\mu}}\,p_o^{\frac{3}{2}}$. Putting $\mu' = \mu\,\dfrac{a^3b^3}{{p_o}^3}$, the periodic time

$$= \frac{2\pi}{\sqrt{\mu}}\,p_o^{\frac{3}{2}} = \frac{2\pi}{\sqrt{\mu'}}\,(ab)^{\frac{3}{2}},$$

and therefore when the force to O

$$= \frac{\mu}{r^2\,(b^2-p_1p_2)^{\frac{3}{2}}},$$

the periodic time

$$= \frac{2\pi}{\sqrt{\mu}}\,(ab)^{\frac{3}{2}}.$$

When O coincides with a focus, $p_1p_2 = 0$; the force to $S = \frac{\mu}{b^3} \cdot \frac{1}{r^2} = \frac{\mu'}{r^2}$ suppose, and the periodic time

$$= \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} b^{\frac{3}{2}} = \frac{2\pi}{\sqrt{\mu'}} a^{\frac{3}{2}}.$$

When O coincides with the centre,

$$p_1p_2 = -a^2e^2 \sin^2 \theta,$$

where θ is the inclination of CP to the major axis, therefore

$$\text{force to O} = \frac{\mu}{r^2 (b^2 + a^2e^2 \sin^2 \theta)^{\frac{3}{2}}}$$

$$= \frac{\mu}{r^2 (a^2 \sin^2 \theta + b^2 \cos^2 \theta)^{\frac{3}{2}}}$$

$$= \frac{\mu}{r^2} \cdot \frac{r^2}{a^3b^3} = \frac{\mu}{a^3b^3} r = \mu' r,$$

and the periodic time

$$= \frac{2\pi}{\sqrt{\mu}} (ab)^{\frac{3}{2}} = \frac{2\pi}{\sqrt{\mu'}}.$$

§ 14. The substance of the above paper was communicated to the Royal Astronomical Society on December 14, 1877, and an abstract appeared in the account of that meeting in the *Observatory*, vol. i., p. 268 (January 1878); but I have been prevented by other occupations from writing out the paper till now. The theorem that if a body moves under the action of a central force $\frac{\mu r}{p^3}$ the periodic time is $\frac{2\pi}{\sqrt{\mu}} p_o^{\frac{3}{2}}$ was set by me in the Mathematical Tripos, Wednesday morning, January 2, 1878 (see *Solutions of the Senate-House Problems and Riders for* 1878, pp. 41–43).

Trinity College, Cambridge,
1878, *August* 29.

On a Portable Star Finder for Altitude and Azimuth Telescopes.

By Edward H. Liveing, A.R.S.M.

The following is a description of an instrument that I invented in the early part of last year, and that I have had in actual use about twelve months.

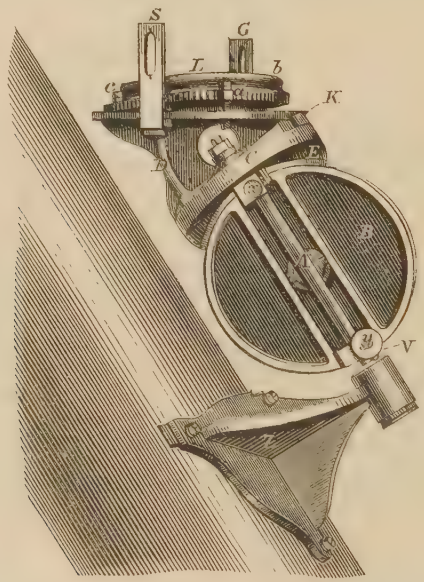
My object in describing it is that I think it may be very useful with the altitude and azimuthly mounted reflectors so much in use among amateurs and others at the present day.

It differs from an ordinary Equatoreal in the facts that it weighs only a few ounces, and that, instead of carrying the telescope, it is itself carried by it, being employed only to direct the telescope, not to support it.

It requires no fixed or adjusted stand. The telescope and stand can be taken in and out of the house at will, and are always ready for use at once.

It enables the time to be ascertained when the Sun or any other celestial object (whose declination and R.A. are known) is visible: this it does by taking an altitude and giving the hour-angle mechanically, without calculation.

I will now, with the aid of the figure, describe its construction.



A is a spindle attached to a telescope by a bracket Z, the spindle being parallel to the optical axis of the telescope;* this spindle carries the circle B, which forms the declination circle of the instrument. This circle is movable around the spindle in the bearings x and y, but its plane remains parallel to the spindle, and therefore to the telescope. Upon the flat surface of this circle moves a brass plate E, carrying at one end a vernier V to read the graduated edge of the declination circle, and at the other the hour circle C of the instrument. Upon the surface of C turns the colatitude bracket D, carrying a vernier K to read the hour circle, and also a circular spirit level L, and two sights,

* See mode of adjustment at end of Paper.

S and G, clamped upon its edge and capable of being secured in any desired position.

Now, before the instrument can be used to find objects, an azimuth mark must be fixed upon, and, for simplicity of description, we will suppose some distant well defined terrestrial object to be visible from the situation where the telescope stands.

Then, to fix the sights, proceed thus:—Set the declination vernier to the declination of the Sun at the time; now direct the telescope to the Sun, so that that object is in the centre of the field; next render the circular spirit level true by moving the hour circle motion, and also the whole instrument about the spindle A. This done, adjust and clamp the sights so as to bisect the distant object fixed upon;* now read the hour circle, which gives the hour angle of the Sun at the moment, which, added to the Sun's R.A. = the sidereal time. To this having set a clock or watch, we are ready to find any object with the instrument in the following manner:—

Set the declination vernier to the declination of the object and the R.A. vernier to its hour angle (that is the sidereal time − the right ascension); now render the level correct by moving the telescope in altitude, and also the instrument on the spindle A; this done, move the telescope in azimuth until the sights bisect the distant object, and if all is done right the star will be in the field.

And conversely, if it be required to determine the R.A. and declination of a body for identification, proceed thus:—turn the telescope on the object, noting the sidereal time; move the instrument so that the level is true and the azimuth object bisected by the sights; read the declination circle, which gives the declination of the object, and the R.A. circle, which gives the hour angle of the object, which − the sidereal time = its R.A.

Now supposing no distant terrestrial object suitable for an azimuth mark to be visible from the situation where the telescope stands, the following method may be employed:—

At any convenient distance and direction fix up horizontally two deal laths, in such a manner that a line joining them passes through the place where the telescope stands. This done, direct the telescope as before described to the Sun, setting the instrument to his declination and moving until level; then fix the sights so that they cut the laths near the middle when looking backwards or forwards through them; now mark the points on

* In fixing azimuth marks in the manner described by aid of the Sun or other celestial object, it is well to choose a time when their altitude is changing pretty rapidly, and not when near the meridian; if, however, the local time is known, this restriction is not necessary, as in that case the R.A. circle may be set to the hour angle of the Sun in addition to setting the declination circle to his declination; indeed, in any case when the time is known, it is well to do this, as it insures increased accuracy in fixing the azimuth marks.

the laths where they cut, and mark off also a rough scale of equal parts on both sides from these central marks. The object of these double azimuth marks is to avoid parallax, and their use is very simple : thus, in the case of finding an object, proceed as before directed so far as regards setting to declination and hour angle and setting the instrument level; then, looking through the sights, move the telescope in azimuth until the central mark on one of the laths is cut; now, looking backwards through the sights, note at how many divisions from the central mark they cut the lath; reduce them to half* the number by moving the telescope in azimuth ; the line of sight is now parallel to the line joining the central marks on the laths.

It will be seen that the telescope can be set down anywhere between the laths and is ready for use at once.

In the above account I have avoided going into detail so as not to confuse the description, but it is desirable I should mention a few points I have omitted.

First as to the dimensions of the instrument: one made with 2-inch circles will, I think, be found the most useful size; circles of this size can be made to read to 2″ of arc; it is of great importance that the declination circle be well graduated.

Secondly as regards the sights employed: I have found the simple form of slit and wire sights, such as are used for compasses, to answer very well; they will read either way nearly equally well, that is with the slit next the eye (the ordinary way of using them) or with the wire next the eye; in the latter case, however, the distance between them must not exceed a few inches, as it renders the field of view so inconveniently small when looked through in the reverse direction. With 3 inches distance between the sights they will read to 4′ or even less without difficulty.†

Of course, if greater accuracy is required, it can be obtained by the use of a small telescope, about 4–5 inches long and $\frac{3}{8}$ in. aperture, resting in Y bearings, which take the place of the sights.

There are certain positions of the instrument where a small reflector or reflecting prism is required to enable the sights to be seen through.‡

I should mention the whole instrument can be made to lift off the steel spindle, so that when not in use it can be kept in its own case, the necessary friction to the motion upon the spindle being given by the clamp screw *y*.

The circular spirit level is adjusted by the three screws *a*, *b*, *c*, which give a certain amount of adjustment in latitude. It could

* That is if the telescope is about equally distant from both laths; if not, so move the telescope in azimuth that the sights cut the laths at an equal number of divisions on the same side from the central marks.

† With 5 inches distance between the sights I have found 2′ the maximum error in directing.

‡ Except when the instrument is attached to the telescope near the open end; there, however, it upsets the balance unless counterpoised, and the parallax is greater, and therefore longer deal laths are required.

be easily made adjustable to any latitude, if desired, by having a suitable movement between the level and R.A. circle.

It may be well to state that the instrument shown in the illustration, must only be considered a temporary one, made for the purpose of trying how the method worked, and in several points I think its construction might be improved.

Lastly, I must not forget to mention two suggestions made by my friend Mr. C. V. Boys, namely, having two R.A. circles, and driving the colatitude bracket upon the upper circle by watch-work placed in the bracket and regulated to sidereal time; to find an object it would then only be necessary to set the second hour circle to the R.A. of the object, as with an ordinary Equatoreal provided with double hour circles.

The other suggestion was the addition of a small graduated circle to the movement about the steel spindle, by which addition it would be possible, using an ordinary position angle micrometer on the telescope, to obtain position angles of double stars &c.—the readings of the small circle on the spindle being added to or subtracted from those of the micrometer to obtain the true position angle of the double star observed.

The following are the various adjustments of the instrument, though most should be performed by the maker:—

(1) To determine if the plane of the declination circle be parallel to the axis of the spindle A:—

Place a spirit level on the flat surface of the declination circle (having removed the rest of the instrument) parallel to the direction of the spindle, and level by moving the telescope in altitude; turn the circle through 180° about the spindle; now, placing the spirit level on the opposite face,* observe if it still shows level; any deviation = twice the error.

(2) To ascertain if the R.A. circle is at right angles to the declination circle:—

Having removed the colatitude bracket D and placed a level on the face of the hour circle C parallel to the face of the plate E, move the spindle A and the declination motion until the level shows true during a complete revolution of the instrument around the spindle A. On removing the level and placing it at right angles to the plate E, it will exhibit if the planes of the two circles are at right angles or not.

(3) To determine the index error of the declination circle:—

The instrument being in the position just mentioned, read the declination vernier; the amount that this reading differs from 90° = its index error.

(4) To determine and correct the index error of the R.A. vernier:—

* This supposes the two faces have been turned up parallel to each other.

The spindle A being still as in the last adjustment, truly vertical, put on the colatitude bracket D, and move the R.A. and declination motions till the circular level is true, then the R.A. circle should read 0^h; if not, adjust the level by the screws *a* and *b* until it does.

(5) To adjust the level to the angle of colatitude of the place of observation:—

The spindle as before being vertical, the R.A. vernier at 0^h, and the declination vernier set to the colatitude, adjust the level true by the screw *c*.

(6) To set the spindle parallel to the optical axis of the telescope:—

Set up a scale of equal parts vertically at a distance of 200 yards or so from the telescope, and having placed a spirit level on the surface of the declination circle, parallel to the direction of the spindle (as in the 1st adjustment), move the telescope till it shows level; read scale through telescope; now reverse the telescope, level again as before, note reading of scale again; the midway point between these two readings is the level point; to which having set the telescope, adjust the spindle level by the screws that hold it; now support the telescope on temporary supports the same height as its stand, with the line joining its trunnions vertical, and having directed it to the level point on the distant scale, adjust the spindle level in this direction also.

Note upon the Star Bradley 2935. By Prof. T. H. Safford.

(*Communicated by the Astronomer Royal.*)

The star in question, the *preceding* of a double found in Struve's Dorpat Catalogue, has usually been identified with the *following* component; and this because its proper motion for 100 years is approximately equal to the distance between the two stars.

How the case stands can be readily seen by reducing the extant positions of the preceding star to 1855·0. With the approximate position for that date,

$$\overset{h\ m\ s}{22\ 3\ 11{\cdot}2} \text{ and } \overset{\circ\ '\ ''}{+82\ 10\ 13},$$

we obtain the following formulæ for reduction (by precession only) from 1855 to $1855+t$:—

In R.A. $$-1{\cdot}67298\,t-0{\cdot}3972\,\frac{t^2}{200}-2{\cdot}017\left(\frac{t}{100}\right)^3,$$

In Decl. $$+17{\cdot}5053\,t-0{\cdot}1274\,\frac{t^2}{200}-0{\cdot}511\left(\frac{t}{100}\right)^3,$$

and thus the following positions for the equinox of 1855·0 :—

Authority.		R.A. h m s	Decl. ° ′ ″	O—C s	O—C ″	No. Obs.
Bradley	1755	22 3 17·72	82 10 17·0	+0·06	+1·1	2.3
Groombridge	1807·8	13·96	14·7	+0·44	+0·8	
Struve	1815·2	13·79		+0·14		1
Struve	1823	13·52	14·6	−0·09	+0·1	6
Schwerd	1828	13·2	15·9	−0·09	−1·4	5
Radcliffe	1844·0		13·0		+0·6	6
Armagh	1844·8		16·1		−2·5	1
Radcliffe	1847·8	12·06		−0·22		3
Armagh	1853·9	9·9		+1·55		1
Carrington	1856·6	11·5	14·1	−0·23	−1·1	3
Yarnall	1866·4	10·58	11·3	+0·07	+1·2	4.2
Airy	1867·7	10·69	13·8	−0·12	−1·3	2
Main	1872·8	10·18	11·1	+0·06	+1·1	4.3

The formula for position and proper motion with which comparison is made is

$$\text{R.A.} = 22^{h}\ 3^{m}\ 11{\cdot}38^{s} - 0{\cdot}064^{s}\,(t-1855),$$

$$\text{Decl.} = 82^{\circ}\ 10'\ 13{\cdot}1'' - 0{\cdot}05''\ (t-1855).$$

The systematic corrections, uncertain in this region, have been applied only for Struve 1815, where the observations are arranged purposely for them ; it is at once plain that the following star, which is in about $6^{s}{\cdot}6$ greater right ascension and $3''{\cdot}3$ greater declination, cannot be identical with the one observed by Bradley.

In consquence of the error in identification the proper motion is given

	s	″
By the B.A.C.	+0·017	−0·03.
By Mädler	−0·0101	−0·002.

The latter authority rejects Struve's place of 1823—which I give as used in his discussion, not having the *Positiones Mediæ*—as it does not harmonise with his wrong authorities of later date. The true star is Groombridge 3707, the false one 3709. Slight changes will be necessary in the nomenclature and positions of the Seven-year Catalogue of 1864. The star's proper motion in arc is about $0''{\cdot}14$, in a direction not very far from diametrically opposite to that which the solar motion would give it.

Williams College, Williamstown, Mass.,
June 27, 1878.

On the Results of Meridian Observations of the Mars Comparison Stars. By David Gill, Esq.

Before leaving England for Ascension I applied to all the principal Observatories for their co-operation in observing the Stars I was about to employ in the observations of *Mars*. I would now desire most gratefully to acknowledge the hearty response with which my appeal has been met by most of the principal Observatories, and to tender to the Directors of those institutions my warmest thanks for the invaluable assistance they have thus rendered to me.

I have still to receive the results of a long series of observations made in 1877 and in the present year by Professor Lewis Boss at the Dudley Observatory, as well as the results of observations made in 1877 at Sydney and Liverpool. In addition to this there is the heliometric triangulation, the discussion of which cannot be satisfactorily attempted till all the data depending on meridian observations have been combined and discussed. From twelve Observatories the results have been already received, and for my present purposes I have discussed them in the following paper.

Some unexpected discordances have been found in the re-results of meridian observations at the various Observatories. The explanation of these has raised some interesting and important questions in practical astronomy; and it is in the hope of drawing attention to the subject that I present this paper in its present form.

The following Tables give the immediate results of the observations made at the various Observatories; the small figures below each result indicate the number of observations.

Right Ascensions at the various Observatories depend on the following authorities for the places of the clock stars:—

Königsberg	Verzeichnisses I., *Vierteljahrsschrift der Astron. Gesells.*
Melbourne	*Nautical Almanac* stars, with corrections of $-0^s{\cdot}11$ to α *Columbæ* and $-0^s{\cdot}07$ to *Sirius*, besides additional stars selected from the same Greenwich General Catalogue (1860).
Pulkowa	Verz. I., *Viert. Ast. Gesells.*
Leipzig	Do.
Greenwich	The new Nine-Year Catalogue (Greenwich clock stars).
Berlin	*Nautical Almanac* and Greenwich clock stars.
Leiden	Verz. I., *Viert. Ast. Gesells.*
Paris	Clock stars of Paris Observatory.
Washington	American Ephem. and Washington clock stars.
Cambridge, U.S.	Star places on p. 164, vol. ix., *Proceedings, American Academy*, with a correction, $-0^s{\cdot}016$, to reduce to the equinox of the Verz. I., *Viert. Ast. Gesells.*
Cordoba	American Ephem.
Oxford	*Nautical Almanac.*

Meridian Observations of Mars Comparison Stars, 1877.

Right Ascensions.

	Königsberg.			Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge, U.S.	Cordoba.	Oxford.
	h	m	s	s	s	s	s	s	s	s	s	s	s	s
a	22	47	0·022	·027	*·980	·089	·020	*·973	·090	·014	—	·056	·162	·037
			5	3	3	4	10	4	12	5		6	5	7
b		47	37·753	·803	·791	·748	·784	·780	·866	·908	·917	·892	·854	·761
			6	3	6	3	5	4	4	5	3	6	5	7
a		51	39·420	·577	·464	·390	·550	·505	·648	·456	·660	·767	·634	·408
			7	3	5	3	5	2	4	5	3	3	5	5
c		53	6·920	·910	·918	·937	·920	·888	·986	·982	—	·980	·951	·954
			6	3	5	2	5	4	7	6		6	7	8
β		56	5·544	·487	·522	·545	·526	·580	·641	·540	·687	·628	·572	·463
			6	3	6	4	5	3	2	5	3	6	5	6
μ		58	2·943	—	·975	·930	·932	·897	*·008	—	*·063	—	·960	·914
			6		5	2	5	3	2		3		4	5
d		59	27·942	·880	·880	·780	·844	·800	·913	·878	·973	·968	·888	·817
			6	3	3	1	5	2	3	6	3	6	5	4
γ	23	0	31·624	·553	·545	·580	·595	·590	—	·577	·720	·677	·620	·530
			5	3	2	2	6	1		3	3	6	5	4
e		0	57·928	·875	·954	·940	·912	·920	·957	·730	·987	*·030	*·012	·810
			4	4	5	2	5	3	4	1	3	6	5	3
δ		5	3·872	·853	·862	·610	*·010	·915	*·021	—	*·070	·993	·974	·890
			5	3	5	2	4	2	3		3	6	5	3

Meridian Observations of Mars Comparison Stars, 1877—continued.

Right Ascensions.

	Königsberg.	Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge, U.S.	Cordoba.	Oxford.
	h m s	s	s	s	s	s	s	s	s	s	s	s
$**\epsilon_o$	5 33·468 6	·413 3	·543 3	·700 1	·496 6	·463 3	·586 1	—	·647 3	·658 6	·564 10	[·342]†
θ	7 21·—	—	—	·590 1	·592 5	·515 2	—	—	—	—	—	—
f	8 15·492 5	·418 4	·475 2	·290 1	·395 4	·380 2	—	·329 2	·640 3	·507 6	·498 5	·470 2
g	8 55·935 4	·830 3	·902 6	·755 2	·913 4	·840 5	·949 4	—	*·003 3	·933 6	·890 5	·740 1
$\psi_1 Aq.$	9 26·810 4	·785 4	—	·760 1	·818 5	·715 2	·794 2	·620 1	·890 3	·830 6	·870 5	·808 4
h	11 14·822 5	·740 3	·820 4	·758 2	·728 5	·725 4	·831 2	·760 3	·847 3	·832 5	·752 5	·664 5
i	12 33·740 3	·729 4	·767 3	·705 2	·690 9	·683 3	·759 1	·690 1	·837 3	·769 6	·758 5	·612 5
k	12 37·030 4	·033 3	·063 3	·025 2	·050 4	*·990 3	·089 4	—	·140 3	·170 6	·044 5	*·883 3
l	14 28·600 7	·547 3	·565 4	·503 3	·552 5	·560 5	·658 2	·537 3	·710 3	·642 6	·552 5	·468 4
λ	16 37·534 5	·460 3	·457 3	·340 2	·488 5	·457 3	·575 2	·413 3	·673 3	·555 6	·490 5	·365 4

ζ	16	54·365 4	·383 3	·350 4	·285 2	·404 5	·327 3	·414 3	—	·457 3	·503 6	·430 5	·283 3
η	20	13·325 8	·307 3	·320 3	·240 1	·364 5	·290 3	·419 3	·230 4	·490 2	·437 6	·368 5	·207 7
m	21	41·066 7	·080 3	·118 5	*·940 3	·100 5	·037 3	·144 4	·050 4	·200 3	·080 6	·132 5	*·986 7
n	22	38·841 8	·840 4	·883 3	·920 1	·829 7	·790 4	·869 2	·750 1	·953 4	·925 6	·884 5	·771 8
q	26	3·370 7	·397 3	·390 5	·308 2	·413 4	·345 4	·451 7	·376 5	·527 3	·458 6	·425 4	·363 9
r	29	5·707 7	·657 3	·653 4	·720 2	·676 5	·653 3	·733 4	·623 3	·830	·782 6	·710 5	·674 7
s	29	39·281 7	·280 3	·320 3	·320 1	·276 5	·255 4	·314 2	·230 2	·447 3	·328 6	·350 6	·149 7
t	31	51·314 7	·300 3	·350 1	·173 3	·282 6	·245 4	·347 7	·233 6	·437 3	·315 6	·304 5	·250 7

** By ϵ_0 is to be understood the mean of the two components of this double star.

† It is not mentioned which component of the double star ϵ was observed.

Meridian Observations of Mars Comparison Stars, 1877—continued.

Declinations.

	Königsberg.	Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge, U.S.	Cordoba.	Oxford.
	° ′ ″	″	″	″	″	″	″	″	″	″	″	″
a	−12 16 11·98 5	11·54 6	12·79 3	12·68 4	11·34 9	12·65 4	11·92 12	11·43 3	—	12·34 6	11·90 5	12·10 9
b	−12 50 33·96 5	33·96 4	34·65 6	34·97 3	33·57 5	35·30 4	34·18 4	34·28 4	34·23 3	34·33 6	33·26 5	34·95 7
a	−11 47 21·63 6	20·94 3	21·09 5	22·30 3	20·55 5	21·30 2	20·43 3	19·67 3	21·47 3	20·40 3	20·50 5	20·80 6
c	−13 43 45·66 5	45·57 7	46·66 5	47·72 2	45·15 5	46·84 5	45·92 7	46·11 7	—	46·35 6	45·26 7	46·51 8
β	−11 55 33·93 6	34·26 4	34·75 6	35·16 4	33·92 5	35·30 3	34·99 2	33·53 4	35·57 3	34·09 6	34·06 5	35·13 5
μ	−12 50 28·26 5	—	28·39 5	29·08 2	28·07 5	28·70 3	28·20 2	—	28·37 3	—	27·22 4	28·65 6
d	−11 6 1·37 4	3·24 3	3·83 3	3·74 1	2·84 5	4·45 2	3·64 3	2·80 3	4·76 3	3·83 5	2·32 5	4·84 4
γ	−13 23 27·14 5	26·96 4	28·50 2	27·61 2	26·57 5	28·40 1	—	27·60 3	27·93 3	27·55 6	26·60 5	27·69 4
e	−12 28 14·70 3	15·34 4	15·39 5	17·15 2	14·88 5	15·93 3	14·68 4	14·00 1	16·67 3	15·48 6	15·18 5	14·98 4

δ	−11	10	31·78 (5)	32·24 (4)	32·75 (5)	31·90 (2)	31·12 (4)	33·00 (2)	31·87 (4)	—	33·96 (3)	31·95 (6)	31·50 (5)	32·59 (4)
ϵ_2	−12	36	0·90 (5)	0·95 (3)	1·82 (3)	0·73 (1)	1·46 (5)	2·50 (3)	1·49 (1)	—	2·02 (3)	1·78 (6)	0·87 (10)	[2·94] (4)
θ	−14	3	—	—	—	47·84 (1)	46·10 (5)	46·70 (2)	—	—	—	—	—	—
f	−11	21	24·70 (5)	25·20 (3)	24·91 (2)	25·84 (1)	24·70 (4)	26·50 (2)	—	26·35 (2)	26·16 (3)	25·29 (6)	25·06 (5)	25·33 (4)
g	−12	14	4·53 (4)	3·76 (3)	4·59 (6)	4·49 (2)	4·63 (4)	5·98 (5)	4·74 (5)	—	6·10 (3)	5·61 (6)	4·70 (5)	5·37 (3)
ψ_1 *Aq.*	− 9	45	26·40 (3)	26·80 (3)	—	27·30 (1)	26·80 (5)	27·10 (2)	27·01 (2)	28·20 (1)	27·07 (3)	27·69 (6)	26·76 (5)	27·44 (4)
h	−12	23	3·12 (4)	3·88 (3)	4·08 (3)	4·89 (2)	3·40 (6)	4·98 (4)	3·93 (2)	4·70 (3)	3·90 (3)	4·44 (5)	3·52 (5)	4·43 (6)
i	−11	16	56·10 (4)	57·60 (3)	59·27 (3)	58·17 (2)	57·28 (9)	58·73 (3)	57·37 (1)	58·70 (1)	59·57 (3)	58·43 (6)	57·32 (5)	58·46 (9)
k	−12	50	34·10 (3)	32·93 (3)	34·29 (3)	33·96 (2)	33·00 (4)	34·30 (3)	33·56 (4)	—	33·33 (3)	33·85 (6)	33·28 (5)	34·21 (3)
l	−11	12	18·70 (5)	19·12 (3)	19·74 (4)	20·46 (3)	19·76 (5)	20·36 (5)	18·96 (2)	19·25 (2)	21·11 (3)	19·65 (6)	18·80 (5)	17·97 (5)
λ	−11	26	49·70 (4)	50·56 (3)	50·93 (3)	51·79 (2)	50·56 (5)	51·93 (3)	50·77 (2)	51·35 (4)	52·40 (3)	50·92 (6)	50·32 (5)	50·87 (6)

Meridian Observations of Mars Comparison Stars, 1877—continued.

Declinations.

	Königsberg.	Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge, U.S.	Cordoba.	Oxford.
	° ′ ″	″	″	″	″	″	″	″	″	″	″	″
ζ	−10 3 32·18 4	32·45 3	34·34 4	32·99 2	32·26 5	33·63 3	32·99 4	—	34·12 3	32·76 6	32·92 5	34·03 4
η	−10 42 36·02 5	36·25 3	37·92 3	38·21 1	36·70 5	37·07 3	37·13 3	37·00 3	38·33 3	36·73 6	37·42 5	36·73 7
m	−12 7 32·80 6	32·60 4	32·87 5	32·84 3	32·70 5	34·00 3	32·89 4	33·65 4	34·37 3	33·62 6	32·92 5	33·73 8
n	− 9 56 32·42 5	32·61 4	34·90 3	33·15 1	32·78 6	33·58 4	33·63 2	34·80 1	33·89 4	33·37 6	32·12 5	33·85 9
q	−11 40 38·70 5	38·58 3	39·21 5	40·07 2	38·88 6	39·80 4	39·30 7	39·30 5	39·67 3	39·85 6	38·45 4	39·53 9
r	−11 14 4·06 5	4·59 3	4·97 4	5·42 2	4·51 5	5·77 3	4·83 4	5·30 3	6·12 3	5·33 6	4·28 5	4·80 7
s	− 9 26 40·66 5	41·00 3	42·34 3	42·03 1	41·60 5	43·23 4	42·32 2	42·15 2	42·25 3	41·77 6	41·76 5	42·34 7
t	− 9 18 27·24 5	27·94 3	29·09 1	28·12 3	27·78 5	28·78 4	27·97 7	28·32 6	29·71 3	28·50 6	27·54 5	28·70 7

I have not stated the authorities for the declinations, as I have not received definite information on all points. I shall merely discuss the declinations to settle the differences of declination, and leave the question of absolute declination for after discussion.

It is necessary, however, to mention here that I have received a letter from Dr. Gould, in which he says: "My latitude determination from five years' observations of circumpolars has lately been brought to completion, and results in a correction of $0''.4$ numerically additive to the declinations communicated; if not too late I will ask you to increase all my declinations by that amount." Dr. Gould's declinations are, however, printed as originally received, but the correction will be taken into account in the final discussion.

The first step was the discussion of the old observations for proper motion. All the existing observations of the stars to be found in Bradley, Piazzi, Lalande, and Bessel were reduced to 1877·0, and in all doubtful cases every existing observation in Argelander, Struve, Airy (Cambridge), Taylor, Pond, the old Greenwich Catalogues, in the Washington Catalogue, and in the Catalogues of Santini was reduced also to 1877·0. I am indebted to Mr. Sadler for searching the Paris observations for recent observations of Lalande's stars.*

Lalande's observations were all reduced from the original *Histoire Céleste* of Lalande, by aid of Von Asten's Tables.

The following systematic corrections to the various Catalogues, founded chiefly on the authority of Auwers in the declinations, and of Newcomb in the right ascensions, were employed :—

	R.A. s	Decl. "
Bradley	±0·00	±0·00
Piazzi	+0·14	−2·30
Lalande	+0·14	−2·30
Bessel, 1825	+0·02	+0·90
Argelander, 1830	+0·02	−0·61
Cambridge (Airy)	−0·04	−0·54
Pond, 1830	−0·05	−1·51
Taylor, 1835	−0·03	−0·37
Green. Cat. 1840	+0·09	+0·69

Only two of the stars were found to have a proper motion whose effect it would be important to take into account, and of these one (that of ψ_1 *Aquarii*) was found to agree with the proper motion of the Greenwich Catalogue. The other proper

* In the Paris observations the apparent places of these stars only are published, and to find a star it is necessary to search the records for many years. When a star has been found there still remains the reduction to mean place. Let us hope that ere long these useful observations will be printed in a more accessible form. I understand from Admiral Mouchez that this work is to be undertaken at once.

motion is that of the star λ, and it is rather remarkable for so faint a star (8th magnitude); it is shown by the following Table:—

	Epoch.	R.A. 1877·0. h m s	Obs.	Decl. 1877·0. ° ′ ″	Obs.
Piazzi	1800	23 16 35·69	4	−11 27 9·09	4
Lalande	1800	35·21	1 only 1 wire	6·49	1
Bessel	1825	36·21	2	5·38	2
Taylor	1835	36·52	3	0·76	3
Washington	1860	36·92	4	26 56·99	3
Santini	1860	36·97	2	58·99	2
Observation in 1877		37·50	44	50·97	46

The following Table gives the actual results for all the stars:—

Proper Motions and Magnitudes of the Mars Comparison Stars.

	Annual Proper Motion In R.A. s	In Decl. ″	Mag.		Annual Proper Motion In R.A. s	In Decl. ″	Mag.
a	±0·000	0·00	6·0	ψ_1 *Aq.*	+0·024	−0·02	4·5
b	·000	+0·06	6·7*	*h*	·000	0·00	6·2
a	− ·002	−0·03	9·5	*i*	·000	0·00	5·2
c	− ·001	0·00	6·0	*k*	− ·005	0·00	7·5
β	− ·008	−0·01	7·7	*l*	·000	−0·02	7·5
μ	·000	−0·04	[8·0]	λ	+0·023	+0·26	8·0
d	·000	−0·03	7·5	*ζ*	·000	0·00	8·0
γ	·000	−0·05	8·5	*η*	− ·004	0·00	8·7
e	− ·004	0·00	7·5	*m*	·000	0·00	6·8
δ	− ·005	−0·05	9·0	*n*	− ·008	0·00	6·5
ϵ_o	− ·004	0·00	8·1	*q*	·000	0·00	7·0
θ	·000	0·00	[7·0]	*r*	·000	0·00	7·0†
f	·000	0·00	7·0	*s*	− ·006	0·00	7·5
g	− ·001$_8$	−0·03	7·0	*t*	·000	−0·04	6·5

The mean epoch of the preceding observations is 1877·80. All the observations were made in August, September, October, November, and December of that year, and it is seldom that the mean epoch of the observations of any star at any Observatory

* Bessel, Lalande, and Santini give respectively 7-8, 8, and 7-8 for the magnitude of this star. At Melbourne it is noted Oct. 24 mag. (7) and Oct. 31 (6½); my own recollection confirms these latter estimates.

† Bessel and Santini both estimate this star 8 mag., and it is so in the Berlin Star Maps. It was observed as 7 mag. both Oct. 4 and Oct. 31 at Melbourne.

The magnitudes of *μ* and *θ* are not derived from modern observations; all the other magnitudes are from the excellent magnitude estimations of the Melbourne observations of the *Mars* stars.

differs $\frac{1}{10}$ year from this epoch, and none differ so much as $\frac{3}{10}$ of a year. Hence for the largest proper motion (that of ψ_1 *Aquarii* or λ) the extreme error of assuming the epoch 1877·80 for all observations can never amount to $\frac{1}{10}''$ of arc, and is an insensible quantity in the mean. We may therefore proceed to find the systematic corrections required to reduce each series to a common system of right ascensions and declinations without regarding the proper motions for the present.

Several courses are open; but that which, for reasons afterwards explained, appears to be the best is to assume that the best value of the absolute right ascension and declination will be found from the mean of the results from all the different Observatories, care being taken to ascertain that at each Observatory everything has been done, as far as possible, to eliminate systematic error and secure an absolute result.

It may of course be urged that at a good many Observatories the same system of standard stars was employed, that therefore all the right ascensions are affected by the common systematic error due to that system of right ascension, and that at other Observatories the declinations have no pretensions to be absolute, but are merely derived from differences of declination from the fundamental stars. Practically, however, these arguments are not exact; for even when the same system of fundamental stars and star places has been employed at two Observatories, there are still systematic discordances in the resulting right ascension of the *Mars* stars amounting to $0^s{\cdot}07$ of time, and when different fundamental stars are employed, there are systematic discordances nearly three times greater than can be accounted for by systematic difference in the system of the fundamental stars employed.

It would appear then that there are sources of systematic error peculiar to each Observatory; and it being impossible to say *à priori* which Observatory is least liable to such systematic errors, the proper course with our present data is to adopt the mean system as probably the most accurate.

To find an absolute mean system it is necessary that all the stars should be observed an equal number of times at all the Observatories, and that the observations at each Observatory should be equally good. The mean of the right ascensions and declinations of each star would then be the most probable value of these coordinates.

None of these conditions are completely fulfilled in the case before us. There are, however, 16 stars which have been observed at all the Observatories; so that if we subtract the right ascension and declination of each Observatory's result from the mean result of each of these 16 stars, we shall have 16 values of the systematic corrections, the mean of which will be a close approximation to the true mean corrections. The results of this process for the right ascensions are given in the following Table, and I shall deal with these in the first place.

Corrections of the Right Ascensions to mean for

	Mean.	Königsberg.	Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge.	Cordoba.	Oxford.
	s	s	s	s	s	s	s	s	s	s	s	s	s
b	37·821	+·068	+·018	+·030	+·073	+·037	+·041	−·045	−·087	−·096	−·071	−·033	+·060
a	39·540	+·120	−·037	+·076	+·150	−·010	+·035	−·108	+·084	−·120	−·227	−·094	+·132
β	5·561	+·017	+·074	+·039	+·016	+·035	−·019	−·080	+·021	−·126	−·067	−·011	+·098
d	27·880	−·062	·000	·000	+·100	+·036	+·080	−·033	+·002	−·093	−·088	−·008	+·063
e	57·921	−·007	+·046	−·033	−·019	+·009	+·001	−·036	+·191	−·066	−·109	−·091	+·111
h	14·773	−·049	+·033	−·047	+·015	+·045	+·048	−·058	+·013	−·074	−·059	+·021	+·109
i	33·728	−·012	−·001	−·039	+·023	+·038	+·045	−·031	+·036	−·109	−·041	−·030	+·116
l	28·575	−·025	+·028	+·010	+·072	+·023	+·015	−·083	+·038	−·135	−·067	+·023	+·107
λ	37·484	−·050	+·024	+·027	+·144	−·004	+·027	−·091	+·071	−·189	−·071	−·006	+·119
η	13·333	+·008	+·026	+·013	+·093	−·031	+·043	−·086	+·103	−·157	−·104	−·035	+·126
m	41·078	+·012	−·002	−·040	+·138	−·022	+·041	−·066	+·028	−·122	−·002	−·054	+·092
n	38·855	+·014	+·015	−·028	−·065	+·026	+·065	−·014	+·105	−·098	−·070	−·029	+·084
q	3·402	+·032	+·005	+·012	+·094	−·011	+·057	−·049	+·026	−·125	−·056	−·023	+·039
r	5·702	−·005	+·045	+·049	−·018	+·026	+·049	−·031	+·079	−·128	−·080	−·008	+·028
s	39·296	+·015	+·016	−·024	−·024	+·020	+·041	−·018	+·066	−·151	−·032	−·054	+·147
t	51·296	−·018	−·004	−·054	+·123	+·014	+·051	−·051	+·063	−·141	−·019	−·008	+·046
Mean		+·004	+·018	−·001	+·057	+·014	+·039	−·055	+·053	−·121	−·073	−·027	+·092

These mean corrections must necessarily be very close approximations to the true systematic corrections, and they may therefore be safely employed to correct the positions of the stars in which observations have been secured at only nine, ten, or eleven Observatories.

Thus the star *a* was not observed at Washington; the correction to the mean result of the eleven remaining Observatories will of course be the mean of the corrections for these Observatories. But the sum of all the corrections is 0; therefore the sum of the corrections for the star *a* will be

$$0 - (\text{Washington correction}),$$

and its mean

$$-\frac{\text{Washington correction}}{11} = \frac{0^s\,121}{11} = 0^s{\cdot}011.$$

The following Table gives the uncorrected means of the incomplete series, with their corrections, and the corrected results so found.

	Places where Observations are wanting.	Uncorrected Mean Sec. of R.A.	Correction.	Resulting Mean Sec. of R.A.
		s	s	s
a	Washington	60·043	+0·011	60·054
c	Washington	6·941	+ ·011	6·952
μ	Melbourne, Paris, and Cambridge	2·958	± ·000	2·958
γ	Leiden	31·601	+ 005	31·606
δ	Paris	3·915	− ·005	3·910
ϵ	Paris and Oxford	33·554	− ·007	33·547
f	Leiden	15·445	+ ·005	15 450
g	Paris	55·881	− ·005	55·876
ψ_1 *Aq.*	Pulkowa	26·791	± ·000	26·791
k	Paris	37·047	− ·005	37·042
ζ	Paris	54·382	− ·005	54·377

Subtracting the result of each Observatory from the mean right ascension so found, we get the following corrections :—

	Königsberg.	Melbourne.	Pulkowa.	Leipzig.	Greenwich.	Berlin.	Leiden.	Paris.	Washington.	Cambridge, U.S.	Cordoba.	Oxford.
a	+·032	+·027	+·063	−·035	+·034	+·081	−·047	+·040	—	−·002	−·108	+·017
c	+·032	+·042	+·034	+·015	+·032	+·064	−·034	−·030	—	−·028	+·001	−·002
μ	+·015	—	−·017	+·028	+·026	+·061	−·050	—	−·105	—	−·002	+·044
γ	−·018	+·053	+·061	+·026	+·011	+·016	—	+·029	−·114	−·071	−·014	+·076
δ	+·038	+·057	+·048	+·300	−·100	−·005	−·111	—	−·160	−·083	−·064	+·020
ε	+·079	+·134	+·004	−·153	+·051	+·084	−·039	—	−·100	−·111	−·017	—
f	−·042	+·032	−·025	+·160	+·055	+·070	—	+·121	−·190	−·057	−·048	−·020
g	−·059	+·046	−·026	+·121	−·037	+·036	−·073	—	−·127	−·057	−·014	+·136
ψ_1	−·019	+·006	—	+·031	−·027	+·076	−·003	+·171	−·099	−·039	−·079	−·017
k	+·012	+·009	−·021	+·017	−·008	+·052	−·047	—	−·098	−·128	−·002	+·159
ζ	+·012	−·006	+·027	−·092	−·027	+·050	−·037	—	−·080	−·126	−·053	+·094
	Mean +·007	+·040	+·015	+·037	+·001	+·053	−·049	+·065	−·119	−·070	−·036	+·051

Combining the results of both tables, having regard to the number of observations in each, we get the following corrections to a mean system derived from all the stars observed :—

Systematic Correction to Mean System of Right Ascensions.

	s		s
For Königsberg	+·005	For Leiden	−·053
„ Melbourne	+·026	„ Paris	+·055
„ Pulkowa	+·005	„ Washington	−·120
„ Leipzig	+·049	„ Cambridge, U.S.	−·072
„ Greenwich	+·009	„ Cordoba	−·032
„ Berlin	+·044	„ Oxford	+·076

I devoted a good deal of time and thought to an endeavour to trace the origin of these systematic discordances, and arrived at the conclusion that they might have three sources:—

1. Difference in the places of the clock stars employed.

2. Errors of pivots producing a change of azimuth, buckling of the cube, or unsymmetrical flexure of the axis in the various transit instruments when directed to different altitudes, producing a change of collimation; the clock stars employed having, as a rule, much greater north declination than the stars observed, a systematic error would be thus created.

3. An error like a personal equation depending on the magnitude of the star.

The reasons which led me to believe that this last was a probable source of error were as follows:—

There appear to be two distinct habits of observing transits with the chronograph, and from conversation with a good many observers, I think the following is the one generally adopted: viz. to endeavour to tap the key at the instant the star is bisected by the transit web; in other words, to give the mental message, to tap just so much in advance that the sound of the tap shall coincide with the instant of bisection of the disk to the eye. Observers unconsciously train themselves to this habit; but Dr. Gould, in his *Transatlantic Longitude*, has shown that it is liable to serious objection. He points out that this method of observing gives rise to a personal equation varying with the declination, the observer of course tapping too soon for slow moving and too late for quick moving stars. For this reason Dr. Gould urges the necessity of the observer waiting until the star appears actually to be bisected, and *then* giving the mental message to the hand to press the chronograph key. In this mode of observing the actual beat of the key of course comes after the star has been bisected, but the interval will always be a constant one so long as the conditions of the observer's nervous system remains unchanged, and the observer's personal equation will

be the same for stars of all declinations. So with stars of different magnitude. If the observer really waits until the star is bisected by the wire, and *then* gives the mental message to press the key, his results will probably be unaffected by magnitude; but if he is in the habit of giving the pressure a short time before actual bisection, he will be nearly certain to press too soon for very bright and too late for very faint stars, because the larger disk and the rings of a bright star will make it appear closer to the web than the small sharp disk of a faint star at the same angular distance from the web.

This appears to be an easy physical explanation of such a personality.

The complete solution of these points I soon found would require special observations for its satisfactory discussion. I accordingly issued a circular to the Observatories that had taken part in the work, to draw attention to this point. I am happy to say the matter has been warmly taken up at several Observatories, and I hope soon to accumulate much useful information on the subject.

I need only at present communicate the following, to show that there does exist a necessity for the study of personal equation depending on star magnitude.

At Leiden Professor Bakhuyzen has made special observations for the purpose. He observed a star alternately with the full aperture of the instrument (16 centimetres), and with the aperture reduced to 6 centimetres, taking precautions to eliminate effects of possible change of collimation which might be produced by applying the diaphragm.

Observer.	R.A. (Aperture 16 c.)−R.A. Aperture 6 c.).		
E. Tr. d. A. S. Bakhuyzen	−0·070	16 stars	5–7 mag.
Wiltendink	−0·025	8	
Stieltz	−0·051	8	

He found that the application of the diaphragm did not produce any sensible change on the collimation.

Assuming that the mean of all the observations at all the Observatories in the *Mars* star series would be free from systematic error depending on magnitude, he obtained the curves which I have given in the accompanying diagram.

Professor Bakhuyzen has made some very interesting researches founded on the results of the observations which I forwarded to him: but as he is about to communicate them to the *Astronomische Nachrichten*, I need not repeat them here. They go to show that for Observatories using the same clock stars there are systematic differences in the right ascensions of the *Mars* stars amounting to $0^{s}\cdot06$ and $0^{s}\cdot07$, and that the extreme difference of the systems of clock stars is $\pm\ 0^{s}\cdot03$.

The importance of the question will be realised when I state that, as the matter stands, a theoretical astronomer receiving

observations of a minor planet from Washington and Oxford would find an inexplicable difference in the right ascension amounting to three seconds of arc.

A good practical instance came under my own experience in the reduction of the *Juno* observations. I could not reconcile the Greenwich meridian observations of right ascension of *Juno* with the Washington and Cambridge, U.S., results; when the latter were fairly represented, the residuals in the Greenwich observations had all the + sign.* The corrections found in the present investigation reduce the whole to order.

The observations I have requested, together with the heliometric triangulation (which is free from the error in question), will go far to settle the question. Meanwhile, I have thought it best to defer the determination of the absolute right ascension to the end, and discuss the observations now on the supposition that each star is affected by a small systematic error besides the accidental errors of observation.

That is to say, if at any Observatory a star is observed n times, the *mean error* (ϵ) of *one* observation will be found by the well-known formula

$$\epsilon^2 = \frac{[vv]}{n-1}, \quad \ldots \ldots \ldots \quad (1)$$

where $[vv]$ is the usual designation of the sum of the squares of the differences from mean, and the mean error of the result of n observations (ϵ_o) will be found by

$$\epsilon_o^2 = \frac{\epsilon^2}{n}. \quad \ldots \ldots \ldots \quad (2)$$

But if the observations at any Observatory are affected by an error which is systematic for each star, as, for example, if the observer's *personal equation* is different for different star magnitudes, then all the observations of each star will be affected by a systematic error due to this habit of observing, an error which will not be eliminated by any number of observations.

If we denote this systematic *mean error* by s, then the true *mean error* (ϵ_1) of each result will be expressed in the formula

$$\epsilon_1^2 = \epsilon_o^2 + s^2 = \frac{\epsilon^2}{n} + s^2. \quad \ldots \ldots \ldots \quad (3)$$

Our problem then is to find the values of ϵ^2 and s^2 for each Observatory in order to compute the true weights of the various results, and so obtain the definitive right ascension of each star.

The first step is to find the mean error of one observation at each Observatory on the supposition of no systematic error. Each star observed more than once at any Observatory gives

* *Dun Echt Observatory Publications*, vol. ii., p. 204.

a value of the square of the *mean error* (ϵ^2) by formula (1); the mean of all the values of ϵ^2 so found from all the stars, having regard to the number of observations, is the true value of ϵ^2 for each Observatory. The resulting values of ϵ^2 are contained in the accompanying Table:—

	ϵ^2	ϵ s
Königsberg	·003110	± ·056
Melbourne	·001790	± ·042
Pulkowa	·005982	± ·077
Leipzig	·012884	± ·113
Greenwich	·006840	± ·083
Berlin	·001391	± ·037
Leiden	·001290	± ·036
Paris	·009644	± ·098
Washington	·004887	± ·070
Cambridge, U.S.	·004457	± ·067
Cordoba	·001046	± ·032
Oxford	·008374	± ·091

Let us now represent by E the true *mean error* of the places which result from the combined observations of all the 12 Observatories. Also for convenience put

α_a, α_b, α_c for the right ascensions of the stars a, b, c, &c., which result from the observations of any particular Observatory; and

α_{oa}, α_{ob}, α_{oc} the right ascensions of the stars a, b, c, &c., derived from the combined results of all the Observatories.

Also put

$$v_a^2 = (\alpha_{oa} - \alpha_a)^2,$$

$$v_b^2 = (\alpha_{ob} - \alpha_b)^2,$$

$$v_c^2 = (\alpha_{oc} - \alpha_c)^2,$$

and v_o^2 = the mean of $(v_a^2 + v_b^2 + v_c^2 + \&c.)$;

that is to say, if there are m stars, adopting the usual notation of the method of least squares,

$$v_o^2 = \frac{[v_o v_o]}{m}.$$

Now if α_{oa}, α_{ob}, &c. were true right ascensions of the stars a, b, &c., then v_o^2 would be the true value of ϵ_1^2 for the particular Observatory under discussion. But as α_{oa}, α_{ob}, &c. are derived from a series of erroneous values of α_a, α_b, &c., made at different Observatories (of which the particular values in question form part), v_o^2 must be smaller than ϵ_1^2, and the true values of ϵ_1^2 become $(v_a^2 + E^2)$, $(v_b^2 + E^2)$, &c. Hence, putting n_o for the mean

number of observations of each star at a particular Observatory, we get the following general equation for the mean error of *one* observation at that Observatory:

$$\epsilon_1^2 = \frac{\epsilon^2}{n_o} + s^2 = v_o^2 + \mathrm{E}^2;$$

and if m stars have been observed,

$$s^2 = \frac{[v_o v_o]}{m} + \mathrm{E}^2 - \frac{\epsilon^2}{n_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This equation is of course only correct for a very large value of m; but as for each Observatory m is = from 21 to 27, we may hope to obtain a good determination of the value of s, provided only that the value of E can be independently determined. From the data of the meridian observations alone the problem is indeterminate, unless we assume that the mean of all the Observatories is affected by no systematic error, or rather that the mean of the systematic errors of the 12 Observatories is the same for all the stars. If this is not the case it is still possible to find afterwards the systematic errors of the mean by comparing the places finally derived from the discussion of the meridian observations with the result of the heliometric triangulation, and this is the course I shall adopt.

Of the 27 stars discussed, there were made in all 1278 observations, or an average of 47·7 observations of each star. The mean of the 12 values of ϵ^2 from the 12 Observatories is ·005135, whence

$$\mathrm{E}^2 = \frac{\cdot 005135}{47\cdot 7} = \cdot 000108.$$

Computing the values of $\frac{[v_o v_o]}{m} + \mathrm{E}^2$, and of $\frac{\epsilon^2}{n_o}$, we get the following Table:—

Square of Mean Error.

	Observed.	Computed.	O−C = s^2
Königsberg	·001824	·000545	·001279
Melbourne	·001134	·000561	·000573
Pulkowa	·001478	·001540	−·000062
Leipzig	·007676	·006212	·001464
Greenwich	·001268	·001300	−·000032
Berlin	·000766	·000437	·000329
Leiden	·000848	·000354	·000494
Paris	·003870	·002737	·001133
Washington	·001091	·001629	−·000538
Cambridge	·002168	·000762	·001406
Cordoba	·001239	·000200	·001039
Oxford	·002525	·001612	·000913

Thus some of the values of s^2 come out with negative signs, and at first it is difficult to see why such an apparent anomaly should be.

Of course, where there is considerable accidental error, but no tendency to systematic error (i.e. if the true value of s is zero), the fact of s^2 coming out a small quantity with a negative sign is merely the result of accident, because our values of v^2, E^2, and ε^2 have been deduced from a limited number of equations, and not from an infinite number as supposed by theory.

But, on the other hand, when s^2 comes out with a considerable negative value—for example, in the case of Washington, when in 22 cases out of 27 the computed error exceeds the difference from mean, there can be no doubt that, for some reason, the computed value of ε^2 was too great.

A little consideration shows that, after all, this should be so, and that in all really good operations the tendency should be to bring out s^2 with a negative sign. In every investigation observations should be so arranged that, as far as possible, all the systematic errors shall alternately affect the final results with reversed signs. The systematic errors should also be determined by independent means and their effects be applied to the results, and from the agreement of these results *inter se* the value of ε be calculated. But if a small error has been made in the determination of any of the constants, the apparent value of ε will be increased, whilst s, being included with opposite signs, has a tendency to disappear in the result.

Take an example. Suppose the collimation of a transit instrument has been reduced, as the observer supposes, to zero, but that as a matter of fact, from yielding of the cube or some other cause, a sensible error still remains. A long series of observations made without reversing may yield very consistent results with a small value of ε, but all affected by a common error s due to constant collimation error. If the observer reverses the transit several times during the observations, the apparent probable error will be increased, because the constant collimation error will enter in among the accidental errors, though in the mean it would be eliminated. In such a case, if the observations were affected by no other systematic error, ε would come out too large, and s^2 would have the negative sign.

This example, however, cannot apply to our case in point, because at Washington the transit instrument was not reversed, and if any error were due to imperfect collimation correction, it would be practically very small, and affect equally all the stars of the zone; it would therefore be included in the systematic correction already applied to all the star places.

But to take a case which is applicable. If different observers are employed at the same Observatory, some of whom have a tendency to observe bright stars too soon relative to faint stars, and others who have the opposite fault, the result would be that the employment of these different observers, whilst it would in-

crease the apparent amount of ϵ, would tend to eliminate s. Hence ϵ being estimated too high, s^2 would come out with the negative sign. To treat the problem rigidly, each observer's results should be dealt with separately, but as the distribution of the observers does not permit this, I am obliged to adopt a less rigid method.

I simply assume that when s^2 has the negative sign its true value is 0, and therefore ϵ^2 has been over-estimated by

$$n_o s^2.$$

Hence, since for Washington $n_o = 3$, we shall have

$$\epsilon^2 = \cdot 004887 - \cdot 001614 = \cdot 003273.$$

Similarly at Greenwich and Pulkowa, where s^2 also comes out negative, the values of ϵ^2 become

	ϵ^2
Greenwich	·006680
Pulkowa	·005734,

the values of n_o being 5 and 4 respectively, the values of s^2 becoming 0 for the three Observatories.

Adopting as the unit of *weight* a *mean error* of $\pm$ 0″·5 of arc on the great circle, that is, of

$$\frac{\pm 0''\cdot 5}{15} \text{ secant } 12^\circ = \pm 0\cdot 0341 \text{ of R.A.}$$

Hence if

$$\text{Weight} = \frac{a}{\epsilon^2},$$

$$a = 0\cdot 001161;$$

and the

$$\text{Weight of any result} = \frac{0\cdot 001161}{\frac{\epsilon^2}{n} + s^2}$$

where the values of ϵ^2 and s^2 may be taken from the preceding Tables for the required Observatory.

In this way the following Table has been computed :—

Right Ascensions.

			Weights for Number of Observations.									
	ϵ^2	s^2	1	2	3	4	5	6	7	8	9	10	11	12
Königsberg	·003110	·001279	—	—	·56	·64	·70	·75	·79	·82				
Melbourne	·001790	·000573	—	—	1·23	1·46	—	—	—	—				
Pulkowa	·005734	·000000	·21	·60	·81	1·25	1·84	2·69	—	—				
Leipzig	·012884	·001464	·09	·18	·27	·36	—	—	—	—				
Greenwich	·006680	·000000	—	—	·60	·84	1·12	1·44	1·80	2·23	2·72	3·31		
Berlin	·001391	·000329	·80	1·54	2·23	2·86	3·46	—	—					
Leiden	·001290	·000494	·86	1·43	1·94	2·37	—	—	3·29	—				4·19
Paris	·009644	·001133	·13	·23	·41	·57	·74	·93	—	—				
Washington	·003273	·000000	—	·69	1·35	2·57	—	—	—	—				
Cambridge, U.S.	·004457	·001406	—	—	·44	—	·56	·61	—	—				
Cordoba	·001046	·001039	—	—	—	1·03	1·08	1·12	1·15			1·21		
Oxford	·008374	·000913	·13	·26	·38	·50	·61	·71	·81	·90	·99			

By a perfectly similar process the Table for the declinations was computed.

The systematic corrections required by the various Observatories to reduce to the mean declination were found to be—

	″		″		″
Königsberg	−0·71	Greenwich	−0·56	Washington	+0·78
Melbourne	−0·49	Berlin	+0·67	Cambridge	+0·09
Pulkowa	+0·36	Leiden	−0·19	Cordoba	−0·60*
Leipzig	+0·40	Paris	+0·01	Oxford	+0·21

* If Dr. Gould's correction to his latitude determination previously referred to had been applied, this correction would become −0″·20.

The *mean errors* of one observation for each Observatory, derived from the agreement of the observations *inter se*, are given in the following Table, together with the value of s^2 deduced as in the right ascensions.

In the declinations the systematic error s probably depends chiefly, if not entirely, on accidental division error (the periodic division error is practically eliminated by the systematic correction applied to all the declinations of each particular Observatory).

Declinations.

	ϵ^2	s^2	Weights for Number of Observations. 1	2	3	4	5	6	7	8	9	10	11	12
Königsberg	1·6710	·0000*	—	—	·45	·60	·75	·90	1·05	1·20				
Melbourne	0·5682	·0000*	—	—	1·32	1·76	—	2·64	3·07	—	—			
Pulkowa	0·6980	·0786	·32	·58	·80	·99	1·15	1·28	—	—	—			
Leipzig	0·6766	·0673	·34	·62	·86	1·06	—	—	—	—	—			
Greenwich	1·0071	·0000	—	—	—	1·48	1·73	1·98	2·23	2·48	2·73			
Berlin	0·2164	·0565	·92	1·51	1·93	2·25	2·51	—	—	—	—			
Leiden	0·2322	·0319	·95	1·69	2·29	2·78	3·19	—	3·84	—	—			4·87
Paris	0·4862	·3585	·30	·41	·48	·52	·54	·56	·58	—	—			
Washington	1·1950	·0464	—	·38	·55	·70	—	—	—	—	—			
Cambridge, U.S.	0·4925	·0217	—	—	1·34	1·72	2·07	2·41	—	—	—			
Cordoba	0·4728	·0503	—	—	—	1·48	1·72	1·94	2 12	—	—	2·56		
Oxford	1·5774	·0037	—	—	·47	·63	·78	·94	1·09	1·24	1·40	1·55		

The values of s^2 marked * come out with a small negative sign, which I have assumed to be the result of accident.

Employing the weights found in the manner described, we obtain the following mean places for the equinox 1877·0 and the epoch 1877·8.

Mean Right Ascensions and Declinations of the Mars Comparison Stars.

For Equinox 1877·0 and Epoch 1877·8.

	R.A. 1877·0.			Weight.	Decl. 1877·0.			Weight.
a	22	47	0·043	10·24	−12	16	12·12	21·11
b		47	37·815	10·53	−12	50	34·33	17·70
a		51	39·549	9·53	−11	47	20·94	14·79
c		53	6·940	9 63	−13	43	46·14	20·02
β		56	5·561	9·87	−11	55	34·57	16·33
μ		58	2·951	7·87	−12	50	28·27	10·84
d		59	27·876	8·92	−11	6	3·53	14·04
γ	23	0	31·601	7·05	−13	23	27·43	12·15
e		0	57·927	9·58	−12	28	15·37	16 03
δ		5	3·929	8·69	−11	10	32·18	15·36
*ϵ_0		5	33·514	8·04	−12	36	1·66	13·97
θ		7	21·580	2·08	−14	3	46·44	3·58
f		8	15·458	7·01	−11	21	25·45	11·70
g		8	55·886	9·60	−12	14	5·03	16·15
ψ_1		9	26·792	7·84	− 9	45	27·21	12·65
h		11	14·766	9·38	−12	23	4·15	15·02
i		12	33·723	9·16	−10	16	58·06	15·33
k		12	37·039	8·83	−12	50	33·70	14·53
l		14	28·579	9·65	−11	12	19·55	15·72
λ		16	37·496	8·88	−11	26	50·97	14·83
ζ		16	54·378	8·95	−10	3	33·09	15·28
η		20	13·340	9·02	−10	42	37·06	15·41
m		21	41·086	10·05	−12	7	33·25	17·55
n		22	38·844	10 07	− 9	56	33·31	16·10
q		26	3·402	10·46	−11	40	39·34	18·29
r		29	5·695	9·72	−11	14	5·01	16·37
s		29	39·293	9·27	− 9	26	42·06	15·06
t		31	51·297	10·05	− 9	18	28·25	17·40

Professor Bakhuyzen has also discussed the right ascensions of these stars independently in a most complete and thorough manner; and if I had no farther material on which to base my final results, I should feel disposed to adopt the results of Professor Bakhuyzen without change. As it is, my own results are

* Mean of the two components, Σ 2988.

only to be regarded as the first step of a discussion in which the heliometric triangulation and the special observations now being made will form part.

It is very interesting, however, to compare the results arrived at by totally independent methods. Professor Bakhuyzen's right ascensions are reduced to the system of the *Viert. Astron. Gesells.* and require the correction $-0^s\cdot020$ to reduce them on the mean to my results. After applying this correction we obtain the following comparison of right ascension :—

Bakhuyzen—Gill.	s	Bakhuyzen—Gill.	s	Bakhuyzen—Gill.	s	Bakhuyzen—Gill.	s
a	+0·007	γ	−0·004	*h*	+0·002	*m*	+0·002
b	− 4	ϵ	− 3	*i*	+ 5	*n*	− 5
α	+ 1	δ	− 4	*k*	± 0	*q*	− 2
c	+ 5	ϵ	− 2	*l*	− 1	*r*	− 3
β	− 2	*f*	− 6	λ	− 3	*s*	− 5
μ	− 4	*g*	− 3	ζ	− 3	*t*	+0·002
d	+0 002	ψ_1 *Aq.*	+0 011	η	−0·006		

The agreement of the two discussions is very satisfactory.

The average weight of a star's right ascension, according to Professor Bakhuyzen, is 162 (in which weight 1· corresponds with a *mean error* of $\pm$ 0^s·111) ; hence the *mean error* in right ascension is $\pm0^s\cdot0087$.

My results give $\pm0''\cdot127$ as the average *mean error* of a star's place in declination. The *probable error*, therefore, is less than $\pm0''\cdot1$ of arc both in right ascension and declination. This result is due to the co-operation of so many Observatories, and the small probable error arrived at refers only to differences of right ascension and declination. The absolute right ascension is uncertain, as yet, to a much larger amount. The conclusions which may be drawn from the discussion appear to be as follows :—

1. In comparing different systems of clock stars, regard is too often paid only to the bright Fundamental Stars, which average second magnitude, whilst the average of the clock stars generally used is 4–5 magnitude. If there is a correction required depending on magnitude (and that appears to be abundantly proved), *the use of bright stars only in such discussions will lead to false results.*

2. There appears to be abundant evidence to show that, besides the errors depending on magnitude, there are others which can only be accounted for on the supposition of instrumental errors. That is to say, after corrections have been applied to reduce the clock stars to a uniform system, then the curves which represent the corrections due to magnitude should pass through $\pm0^s\cdot000$ for stars of 4–5 magnitude (the mean magnitude of clock stars). A reference to the diagram will show that this is not the case.

Now the clock stars have, as a rule, considerably greater north declination than the *Mars* stars; hence a pivot error changing the azimuth and level at different altitudes, or a weakness of one side of the cube, producing a changing error of collimation, which reaches its maximum at the zenith, would produce a systematic difference in the places of the *Mars* stars as compared with the places of the clock stars.

Whilst I admit that these curves result from comparatively limited data, there still appear to be outstanding differences, which can only be accounted for by such errors as I have described. This points to *the necessity of more rigid determination of the errors of meridian instruments, particularly of the change of collimation at different altitudes;* a matter almost entirely neglected at present, and specially necessary in view of the increasing size of meridian instruments and their less frequent reversal.

3. The *personality depending on magnitude* requires more complete discussion. It *should be determined for each observer, and this determination ought to be as rigidly looked to as the ordinary personal equation* is when different observers are employed at the same Observatory.

It can easily be accomplished as follows:—

A bright star is observed over the first five wires; a diaphragm is applied in front of the object-glass (by which the star is reduced a known number of magnitudes in brightness), and the star is then observed over the last five wires. These transits, on being reduced to the middle wire give the personality due to difference of magnitude. Another observation, in which the operation is reversed (the diaphragm being used with the first five wires, the full aperture with the last five), eliminates the possible change of collimation which may be supposed to arise from the application of the diaphragm, and also eliminates errors in the assumed wire intervals.

It was in this way that Professor Bakhuyzen determined for his assistants' observations in right ascension the corrections which I previously quoted. It is somewhat remarkable that for all three assistants a negative correction of their results in right ascension is required for the fainter stars.*

It is still more remarkable, however, if confirmed by more extended experiment, that, as a rule, when chronographic registration is employed, the required correction to the observed right ascension of faint stars is $-$, and when eye and ear observations are employed it is $+$. It is most satisfactory to find that this law coincides with the explanation I attempted to give as

* Argelander employed what is in some respects a better method, viz. the observation of the right ascensions of the variable stars. The method has the advantage that the determination of personality is made in the ordinary course of regular observation, but it can only be employed by an enthusiastic observer carrying on a long and well sustained series of observations with a fixed purpose.

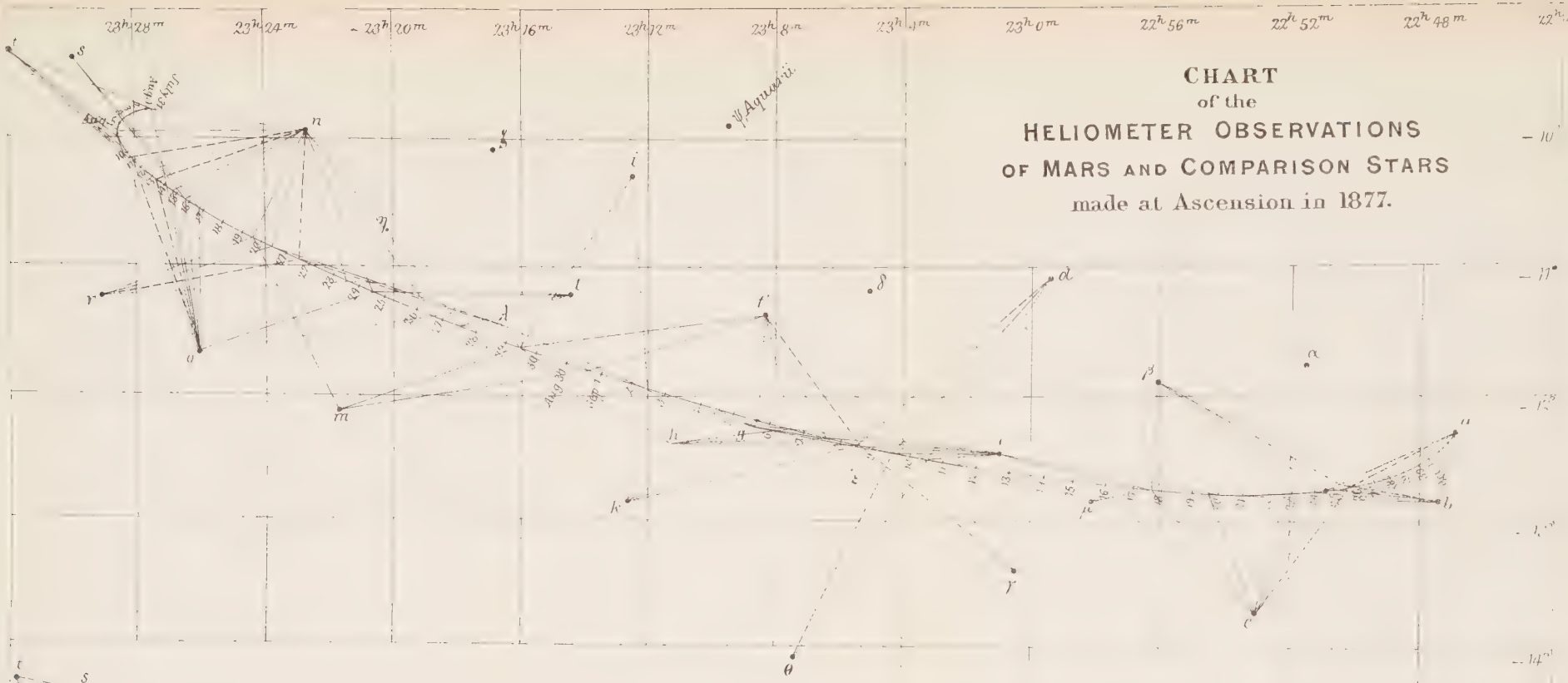
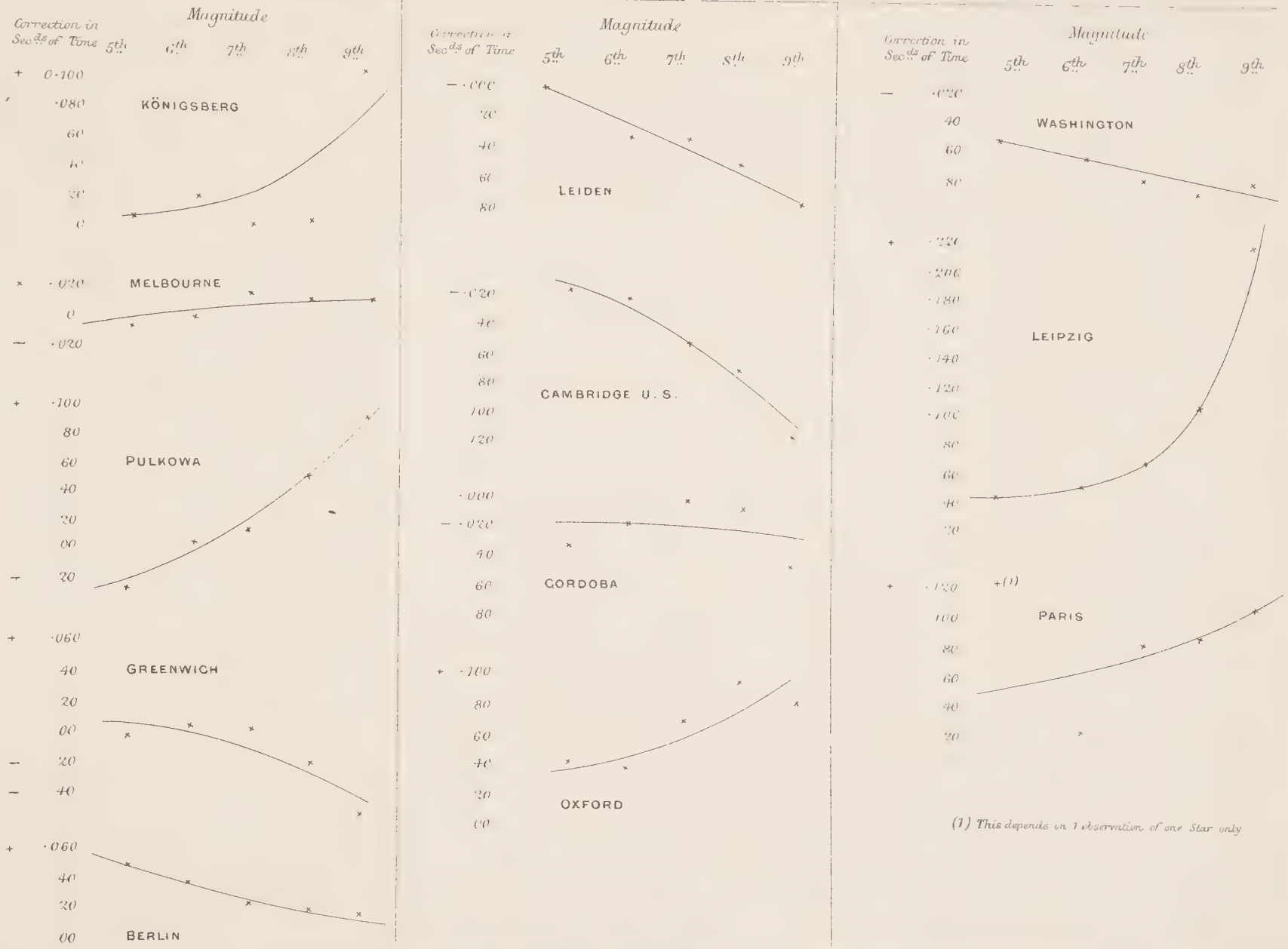


CHART
of the
HELIOMETER OBSERVATIONS
OF MARS AND COMPARISON STARS
made at Ascension in 1877.

CHART
of the
HELIOMETRIC TRIANGULATION
OF THE MARS COMPARISON STARS.

DIAGRAM OF CURVES OF ERROR IN RIGHT ASCENSION DEPENDING ON MAGNITUDE.



(1) This depends on 1 observation of one Star only

Spottiswoode & Co. Lith. London.

to the physical cause of the personality in question, viz. that in chronographic registration the tendency will be to press the key too late for faint stars, because their small neat disks make them appear further from the web than a bright star with its larger disk and rings at the same angular distance from the web.

I think it, therefore, not improbable that there may still remain systematic errors depending on magnitude in the right ascensions which have been deduced in the present discussion. The comparison with the heliometer measures will, however, probably settle the question.

The accompanying diagram shows the measures that have been made in the triangulation, and will enable the reader to form some idea of the geometrical conditions of rigidity which have been realised in the operation.

Another portion of the same plate shows similarly the measures from the stars to *Mars*.

A line thus ——— denotes both Morning and Evening Observations.

A line thus ..... Evening Observations only.

A line thus - - - - - Morning Observations only.

On Observations of α Centauri made with the Heliometer at Ascension in 1877. By David Gill, Esq.

The following observations of the remarkable binary *α Centauri* were made during my stay at Ascension in the latter part of 1877. As I am frequently asked for the results of my measures of this object, I have thought it best to lay them before the Society without further delay.

The observations were not easy, as the following considerations will show.

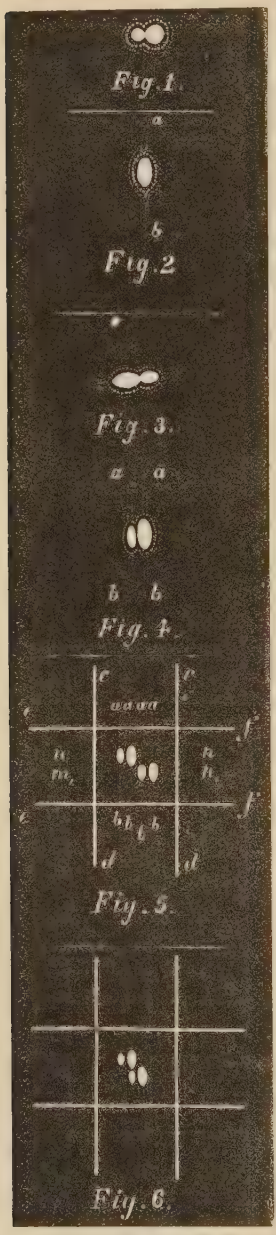
The image of an infinitely distant luminous point is a disk surrounded by rings; the diameter of this disk varies inversely as the aperture of the telescope. For a telescope of 4 inches aperture (that of Lord Lindsay's heliometer) this disk has a diameter of about 1″; in other words, a telescope of 4 inches aperture will barely separate a double star whose distance is 1″, the effect being to produce the appearance in fig. 1.

The half of a heliometer object-glass, however, produces a peculiar image of a single star, somewhat as in fig. 2.

Roughly speaking, it is an ellipse, whose axes are as 2 to 1, the smaller diameter being at right angles to the line of section of the object-glass, and the rings extending into a diffraction ray *ab*.

Now it happened that the first time I turned the heliometer upon *α Centauri* I found it, to all appearance, a single star rather ill defined, looking somewhat like a single star seen in a badly centred object-glass. The fact was, however, that the

heliometer happened to be set so that the position angle of the line of separation of the segments was nearly at right angles to the position angle of the star; the true effect was to produce the appearance shown in fig. 3, and the air being unsteady, the rings were mixed up with the disks in a puzzling way.



When, however, the position circle was set nearly to the true position angle of the stars, the effect became as in fig. 4, and in still moments the diffraction rings were well seen and also the fine diffraction lines *ab*, as in fig. 2.

It is evident that in such an image distances can be more accurately measured than position angles, and accordingly measures of the latter were made in greater number in order to give the results nearly equal accuracy with those from measures of distance.

In position-angle measures the observations were made as follows:—

In the field of view, and accurately adjusted to the plane of the principal focus of the objective, there was a square formed by fine flat gold wires *cd* and *ef* (fig. 5).

The wires *ef* were carefully adjusted to parallelism with the motion of the segments by setting the position circle to 90° + index correction, and allowing a star to run along the wires *ef*, or by placing a star on the edge of a wire and moving the segments rapidly to test if the star ran truly along its edge. This latter could be done almost independently of errors in the clockwork movement when the position circle was set to 90°.

This adjustment having been made, it was tested over and over again by both the methods described without my being able to improve it. I have therefore not attempted to apply any correction, and any error there may be in the coincidence of these wires with the motion of the segments is certainly very far within the accuracy

aimed at in the measure of position angle of so close a double star as α *Centauri*.

In making the measure of position angle the heliometer tube was turned in its cradle till the imaginary lines mn and m_1n_1 (fig. 5) seemed parallel with the lines *ef*. The imaginary lines mn, m_1n_1 are supposed to pass through the centres of the images formed by segment A and segment B of the object-glass respectively. The lines mn and m_1n_1 would of course coincide when parallel if the two segments could be adjusted so perfectly that their optical centres exactly coincided at least distance; that is to say, if the segments were so adjusted that the images of a single star produced by both segments could be exactly superposed by simple motion along the slides. Such an adjustment, even if once made, can never be long maintained, and during the observations at Ascension the error generally amounted to about 2″. Thus the appearance of the stars when measured for position angle was nearly as shown in fig. 5. Of course the segments were reversed in each measure, and after each bisection the setting was displaced and brought up in alternate bisections from opposite directions.

By use of the reversing prism all the observations were made with the stars apparently either vertical or horizontal, and in each set of four bisections (constituting one measure) two bisections were made in each of two opposite directions of the image; that is, two bisections were made with the brighter star apparently to the right or above, and two bisections with the brighter star apparently to the left or below. No systematic difference was found between the position angles measured with the stars vertical and horizontal.

An observation for position angle having been made, the circle was set to the mean of the four readings which constituted the measure in position angle when the stars were arranged as in fig. 5. By careful movement of the slow motion handle, taking advantage of steady intervals of definition, the four diffraction lines were arranged at equal distance from each other and the scales were read off; the segments were then reversed, and the process was repeated. Now the little reversing prism was rotated 180°, the latter operation repeated, and finally the first operation was repeated. In this way the double distance of the stars was measured. When the observations were made in daylight, however, the diffraction lines *ab* could not be seen, and I then thought it more exact to measure the single distance, as in fig. 6, with similar precautions. After all the instrumental corrections have been applied, the following are the results :—

Results of Measures of α_1 and α_2 Centauri at Ascension Island.

Date.	Distance.	No. of Obs.	Position Angle.	No. of Obs.	Remarks.
	"		° ′		
1877·559	2·220	1			
·568	2·128	1	80 31	3	
·592	2·055	1	80 29	2	
·617	...	...	80 38	4	
·628	1·942	2	82 1	4	
·642	...	...	81 49	4	
·800	1·937	2	...		Measures made with great difficulty from unsteady air.
·827	1·863	1	99 41	2	
·841	1·760	3	94 36	3	
·876	1·806	1	99 54	2	
·901	1·438	1½	102 11	2	Definition very bad.

Thus, in hardly more than four months, we have evidence of a change of fully 0″·5 in distance and 20° in position angle. In connection with these measures, it is interesting to hear from Mr. Ellery (*The Observatory* for August 1868) that, from observations on May 8 of the present year, the components are now separating. Obviously, then, the above measures have been made during a critical and important epoch in the orbit. I only regret that much other work prevented my making more numerous measures, including some on artificial stars.

Additions to Memoir on the Theory of the Sidereal System.

By Maxwell Hall, Esq.

There are a few more stars to be added to the lists given in the Memoir on this subject, vol. xliii. of the *Memoirs*, and there are several Notes to be made respecting stars already discussed. These additions are for the most part satisfactory, and confirm the Theory as formerly explained.

It may be remembered that the Sun and the nearer stars were assumed to revolve in circular orbits about their common centre of gravity, a point in space whose position was found to be—

	°	
R.A.	9	15
N.P.D.	63	28

} 1850, Jan. 1·0,

and whose distance is 31 million times the distance of the Earth

from the Sun; that by means of these data, which were obtained by combining the direction of the Sun's motion through space with the proper motions and distances of *α Centauri*, *61 Cygni*, and *Sirius*, the parallaxes and radial velocities of several other stars were computed from their observed proper motions; that the results of the comparison between observation and theory were highly satisfactory in several cases; but that for a few stars the results were decidedly wrong.

This peculiarity still exists, and four of the stars considered give erroneous results, possibly due to the assumed circularity of their orbits.

The following Table contains all the stars whose parallaxes have been found by observation; they are 23 in number; and the differences between observation and computation are exhibited as clearly as possible. The radial velocities are expressed in terms of mean radii of the Earth's orbit per annum; in order to express these velocities in miles per second, they must be multiplied by 2·9.

Star.	M.	Annual Parallax.			Radial Velocity.		
		Obs.	Comp.	O−C.	Obs.	Comp.	O−C.
		″	″	″			
α Centauri	0·6	0·936	0·936	0·000			
61 Cygni	5·5	0·438	0·422	+0·016			
Sirius	0·1	0·204	0·210	−0·006			
Arcturus	0·8	0·133	0·188	−0·055	−15·4	−14·5	−0·9
Vega	1·0	0·098	0·069	+0·029	−15·9	−17·5	+1·6
Procyon	1·0	0·123	0·074	+0·049	+ 9·7	+ 8·6	+1·1
Capella	1·0	0·046	0·029	+0·017	+ 9·7	+ 8·7	+1·0
Altair	1·3	0·181	0·146	+0·035	−13·3	−14·1	+0·8
70 *p* Ophiuchi	4·5	0·160	0·099	+0·061			
34 Groombridge	8·0	0·307	0·195	+0·112			
Polaris	2·3	0·068	0·008	+0·060			
85 Pegasi	6·0	0·050	0·071	−0·021			
3077 Bradley	6·0	0·070	0·104	−0·034			
η Cassiopeiæ	4·0	0·154	0·065	+0·089			
μ Cassiopeiæ	5·5	0·342	0·199	+0·143			
γ Draconis	2·6	0·092	0·007	+0·085			
β Centauri	1·2	?	0·001	...			
21185 Lalande	7·5	0·501	0·753	−0·252			
ι Ursæ Majoris	3·5	0·133	?	...			
1830 Groombridge	6·5	?	2·682	...			
σ Draconis	5·0	0·255	1·963	...			
17415 Oeltzen	9·0	0·247	neg.	...			
21258 Lalande	8·5	0·260	neg.	...			

It will be seen that out of these 23 stars there are two whose computed parallaxes are far too large, and that there are two whose parallaxes are negative; but on the other hand the close agreement between the observed and computed parallaxes and radial velocities of *Arcturus*, *Vega*, *Procyon*, *Capella*, and *Altair* excludes the possibility of such agreement being due to chance; and in forming equations of condition for the more accurate determination of the constants of the system it will be advisable to reject the discordances, and to give great weight to the agreements in both parallax and radial velocity.

But the time has not yet arrived for this redetermination; only six stars out of the 23 have been observed with the spectroscope; and although many of the stars are very small, others are large enough, as *Polaris*, *γ Draconis*, *α* and *β Centauri*, and a few more.

With regard to those stars which have been observed with the spectroscope, but whose parallaxes are as yet unknown, it might seem proper to compute their parallaxes and radial velocities for the sake of the comparison of the latter; but many of these stars have very small and therefore uncertain proper motions, and the few examples given in the memoir show that no great advance would be made. And indeed it will be better to delay all extensive calculations until the constants have been more accurately ascertained.

The following notes and computations refer to the details of the alterations and additions comprised in the above Table; they will be found useful in the reconstruction of the system.

α Centauri.

A valuable paper by Mr. Stone on the proper motions of southern stars will be found in vol. xlii. of the *Memoirs*, where the following are given for this star:—

$$\text{P.M. in R.A.} = -0''\cdot485$$
$$\text{P.M. in N.P.D.} = -0''\cdot75.$$

As these proper motions agree very closely with those adopted in the Memoir, page 168, no recomputation was considered necessary for the present; but, for the future, the proper motions given by Mr. Stone should be employed.

61 *Cygni.*

In the *Observatory* for September 1878 Dr. Ball discusses a series of observations for parallax and finds $0''\cdot465$. This is very satisfactory, and confirms the adopted value of $0''\cdot422$. Dr. Ball also gives seven other determinations; if we include his own and take the mean of the eight, we shall find $0''\cdot438$ as the probable parallax of this star.

Sirius.

The parallax of 0″·193 ascribed to this star by Professor Gylden has been generally adopted since the Memoir was written. We have therefore for this star

	″
Henderson, corrected by Peters	0·15
Abbe	0·27
Gylden	0·193
Mean	0·204

The radial velocity found by Dr. Huggins has been confirmed at Greenwich very recently. By giving the weights indicated in the Greenwich Table of "Motions of Stars in the Line of Sight," *Monthly Notices*, vol. xxxviii., p. 507, the radial velocity observed is +7·9; Dr. Huggins found +6·9; but as no correction can yet be made for the binary motion of *Sirius*, its radial velocity has been omitted in the above Table.

Arcturus.

The parallax of 0″·127 found by Peters was confirmed by the parallax of 0″·138 found by Johnson, *Monthly Notices*, vol. xvii., p. 271; a mean parallax of 0″·133 should be adopted.

Again, the recent observations at Greenwich give a radial velocity of −11·7; Dr. Huggins found −19·0; and the mean, −15·4, agrees very closely with the theoretical radial velocity of −14·5.

Vega.

It was shown that the probable value of the observed parallax was about 0″·09; and this is confirmed by Johnson' parallax of 0″·08, and by Brünnow's recent determination of 0″·13; a mean parallax of 0″·098 should be adopted.

For the radial velocity we have

Dr. Huggins	−17·0
Dr. Vogel	−17·9
At Greenwich	−12·8
Mean	−15·9

The theoretical value is −17·5.

Procyon.

A radial velocity of +9·7 has been observed at Greenwich; the theoretical velocity of +8·6 has therefore been confirmed.

Capella.

As for *Procyon*, a radial velocity of +9·7 has been observed at Greenwich; the theoretical velocity of +8·7 has therefore been confirmed.

Polaris.

A parallax of 0″·091 was adopted in *Nature*, 1876, November 23; and in the same periodical Professor Gylden adopted a parallax of 0″·046, 1878, October 24. And as Peters found 0″·067, the most probable value of the parallax is 0″·068.

It is to be regretted that *Polaris* has not yet been observed with the spectroscope for velocity in the line of sight.

85 *Pegasi.*

A parallax of 0″·05 has been found by Dr. Brünnow for this star; the proper motions used in the computations were taken from the B.A.C.

3077 *Bradley.*

A parallax of 0″·07 has been found by Dr. Brünnow for this star; the proper motions used in the computations were taken from the B.A.C.

η Cassiopeiæ.

A parallax of 0″·154 was assigned to this star in *Nature*; the proper motions used in the computations were found by Main.

μ Cassiopeiæ.

A parallax of 0″·342 has been found by Professor O. Struve; the proper motions, which are very large, have been taken from Main's paper.

γ Draconis.

The observations made at Greenwich gave a negative parallax to this star; a parallax of 0″·092 is given in *Nature*, Chambers's *Descriptive Astronomy*, and elsewhere. Main's proper motions have been employed; they are very small.

β Centauri.

In vol. xxi. of the *Memoirs* there is an elaborate discussion of the parallax of this star by Maclear, who reduced the whole series of observations in N.P.D. to the equation

$$\pi = 0''{\cdot}4817 + 0{\cdot}3987\,\mu,$$

where π is the annual parallax of the star and μ its proper motion in N.P.D., both expressed in seconds of arc.

Maclear found $\mu = +0''.064$ at the commencement of that memoir; but afterwards he adopted $-0''.03$ with no obvious reason, and so deduced a parallax of $0''.47$; at the same time the constant of aberration became $20''.59$.

This determination is unsatisfactory, and the star was not considered in the Memoir on the sidereal system; but as the parallax of $0''.47$ has been adopted by two or three writers, it becomes necessary to include this star.

The proper motions according to Mr. Stone are $-0^s.010$ in R.A. and $+0''.05$ in N.P.D.; and by employing this value of μ, π becomes $0''.502$; the computed parallax is insensible.

21185 *Lalande.*

A parallax of $0''.501$ has been found for this star by Winnecke. The proper motions given by Lynn are $-0^s.044$ in R.A. and $+4''.66$ in N.P.D.

17415 *Oeltzen.*

This star gave a negative parallax, and I threw considerable doubt upon my own work because, in the *Astronomical Register*, vol. xiii., p. 272, it is referred to as 7515 Oeltzen, and a misprint seemed improbable. I have to thank Dr. Krüger for sending me a reprint of his article on the determination of the parallax of this star, which was published in the *Acta Soc. Scien. Fennicæ*, *t.* vii.

The star is Nos. 17415, 6 Oeltzen; and no alteration can be made at present in the former computations.

21258 *Lalande.*

An unfortunate mistake has occurred with respect to this star. Mr. Lynn confirmed the proper motions found by Professor Argelander, and in the *Monthly Notices*, vol. xxxiii., p. 102, he gave $-0^s.386$ in R.A. and $-1''.36$ in N.P.D. This was in December 1872; early in the year 1873, when I was abroad, I adopted his determinations as recorded in the *Astronomical Register*, vol. xi., p. 5, where the negative sign was unfortunately omitted before the proper motion in N.P.D. Consequently it was assumed that the proper motion was positive, and it is not a little remarkable that the erroneous computations gave a correct value of the annual parallax, while the correct computations give a negative parallax, which can only be considered as erroneous in the present state of the theory

No.	Star.	M.	R.A. h m s	P.M. in R.A. s	N.P.D. ° ′	P.M. in N.P.D. ″
1	85 Pegasi	6·0	23 54 21	+0·067	63 43	+0·95
2	3077 Bradley	6·0	23 6 4	+0·201	33 40	−0·28
3	η Cassiopeiæ	4·0	0 40 3	+0·132	32 59	+0·49
4	μ Cassiopeiæ	5·5	0 58 19	+0·386	35 49	+1·56
5	γ Draconis	2·6	17 53 8	0·000	38 29	+0·04
6	β Centauri	1·2	13 53 17	−0·010	149 39	+0·05
7	21185 Lalande	7·5	10 55 8	−0·044	53 2	+4·66
8	21258 Lalande	8·5	10 58 0	−0·386	45 42	−1·36

No.	log l	log m	log n	log δl	log δm	log δn
1	+9·95248	−8·34571	+9·64622	+ 9·64621	+ 9·94959	− 9·93033
2	+9·73165	−9·11145	+9 92027	+ 9·21245	+10·22531	+ 9·19095
3	+9·72924	+8·97630	+9·92367	+ 9·33646	+10·05423	− 9·42611
4	+9·75308	+9·16833	+9·90896	+ 9·56937	+10·55606	− 9·96042
5	−8·27049	−9·79380	+9·89364	− 6·97220	− 8·49551	− 8·39605
6	−9·64818	−9·37962	−9·93599	+ 7·30750	+ 8·94047	− 8·40250
7	−9·88491	+9·34856	+9·77913	−10·18865	+10·11025	−10·57093
8	−9·83864	+9·28163	+9·84411	+10·30600	+10·57276	+ 9·98827

No.	log a	b	c^2	H	K	log (H+$\Pi\rho_0$)	log (K+$a\Pi$V)
1	+9·12070	−1·267	1·714	−0·9861	−0·139	− 9·95075	− 9·11394
2	+9·68843	−1·314	2·873	− ·8292	−0·455	− 9·88395	− 9·62634
3	+9·48940	−1·108	1·402	− ·8617	−0·236	− 9·88047	− 9·33445
4	+9·39005	−3·509	13·917	− ·8835	−0·438	− 9·92952	− 9·62531
5	+9·97598	+0·012	0·002	− ·2436	+0·016	+ 9·84905	+ 8·89763
6	−9·33546	−0·086	0·008	+ ·8127	−0·003	+10·73832	− 8·23045
7	+9·41984	−2·893	17·909	+ ·3768	+2·841	+ 9·58614	+10·45606
8	+9·52035	−2·821	19·020	+ ·2696	−2·759	...	−10·43727

No.	log u	HΠV	log $\left\{\frac{au-b}{+H\Pi V}\right\}$	log ϖ	ϖ ″	ρ_0	$\frac{d\rho}{dt}$
1	+ 9·16319	−0·065	+10·08672	8·85303	0·071	14·03	− 2·04
2	+ 9·74239	− ·055	+10·18441	9·01865	0·104	9·58	− 5·29
3	+ 9·45398	− ·057	+10·05652	8·81490	0·065	15·31	− 4·36
4	+ 9·69579	− ·058	+10·55303	9·29945	0·199	5·02	− 2·49
5	+ 9·04858	− ·016	+ 8·89209	7·85075	0·007	141·01	−15·77
6	− 7·49213	+ ·054	+ 9·14922	7·15429	0·001	700·99	+ 2·18
7	+10·86992	+ ·025	+10·68726	9·87656	0·753	1·33	− 9·85
8	...	+ ·018	...	...	...	...	...

Errata in Memoir, vol. xliii.

Page 168, last line. For "0″·78" read "−0″·78."

.. 176, equation (10). For "$2V\{au-b+H\Pi V(1-\Lambda}$" read "$2V\{au-b+H\Pi V(1-\Lambda)\}$."

.. 179, line 4. For "$\frac{d\rho}{d}$" read "$\frac{d\rho}{dt}$."

„ 185, line 20. For "south, preceding" read "south-preceding."

„ 191, line 3. For "0″·223" read "0″·023."

And in Tables &c., *dele* Star No. 10 and substitute from the Tables in the present article Star No. 8.

Reduction of the North Polar Distances of the Cape Catalogue for 1860 *to Auwers' Standard.* By A. W. Downing, B.A., T.C.D.

When comparing Catalogues together for such purposes as the determination of proper motions, it is of course necessary, where extreme accuracy is desired, to reduce the places of stars given in the different Catalogues to a uniform standard, so that the discordances peculiar to each individual Catalogue may be thus eliminated. The standard usually adopted for North Polar Distances is that given by Dr. Auwers in No. 1536 of the *Astronomische Nachrichten*, and in the present paper I have investigated the corrections which it is necessary to apply to the N.P.D.'s of the Cape Catalogue for 1860 to reduce them to this standard.

The Cape Catalogue has first been compared with the Greenwich Seven-Year Catalogue for 1860. There are 385 stars common to the Catalogues which are available for the purpose, after rejecting those whose places in either Catalogue depends on a single observation, and also two other stars, one passing the meridian very near the horizon of the Cape, and whose N.P.D. in the Cape Catalogue depends on only two observations, the other passing the meridian very near the horizon of Greenwich, the place in the Greenwich Catalogue depending on the same limited number of observations. These 385 stars have been arranged in order of N.P.D. and have then been taken in groups, each embracing about 5° of N.P.D., and, by taking the means of the N.P.D.'s and of the differences between the Catalogues for each group, I get 16 sets of mean differences extending from N.P.D. 49° 51′ to 124° 34′. These differences have been laid down and a curve drawn through the points which may be taken as representing the systematic differences between the Catalogues. The following Table gives the differences as computed and as read off from the curve.

N.P.D.		Number of Stars.	Gr.−C.	
			Computed.	Curve.
°	′		″	″
49	51	4	+0·52	+0·58
57	48	5	+0·43	+0·53
62	43	21	+0·29	+0·50
67	44	36	+0·60	+0·48
72	24	20	+0·33	+0·45
77	29	25	+0·55	+0·38
82	32	27	+0·41	+0·33
87	26	25	+0·14	+0·31
92	30	25	+0·20	+0·38
97	44	31	+0·56	+0·42
102	56	15	+0·24	+0·39
107	35	38	+0·45	+0·33
113	2	25	+0·05	+0·24
117	19	47	+0·26	+0·17
121	33	28	+0·02	+0·12
124	34	13	+0·23	+0·20

The differences have then been read off from the curve for every 4°, beginning from N.P.D. 48°, and, by applying the corrections to the Greenwich N.P.D.'s given in Dr. Auwers' paper before referred to, the corrections applicable to the Cape N.P.D.'s to reduce them to the standard have been obtained. The results are given in the following Table :—

N.P.D.	Gr.−C.	Standard−Gr.	Standard−C.
°	″	″	″
48	+0·59	−0·21	+0·38
52	+0·57	−0·30	+0·27
56	+0·54	−0·39	+0·15
60	+0·51	−0·38	+0·13
64	+0·48	−0·40	+0·08
68	+0·46	−0·44	+0·02
72	+0·45	−0·47	−0·02
76	+0·41	−0·49	−0·08
80	+0·35	−0·41	−0·06
84	+0·31	−0·29	+0·02
88	+0·34	−0·22	+0·12
92	+0·37	−0·18	+0·19
96	+0·41	−0·16	+0·25
100	+0·41	−0·19	+0·22
104	+0·37	−0·29	+0·08

N.P.D. °	Gr.−C. ″	Standard−Gr. ″	Standard−C. ″
108	+0·30	−0·36	−0·06
112	+0·26	−0·47	−0·21
116	+0·20	−0·36	−0·16
120	+0·13	−0·38	−0·25
122	+0·13	+0·9	+1·0
124	+0·18	+1·4	+1·6

For the southern stars whose N.P.D.'s are greater then 120° I have compared the Cape Catalogue with Henderson's Catalogue of 172 stars published in the 10th volume of the *Memoirs*. Dr. Auwers uses the latter as a provisional standard for southern stars, as it appears from his investigation that, up to the zenith of the Cape, it does not differ much from his standard. The epoch of Henderson's Catalogue is 1833·0, and the places of the Cape Catalogue have been brought back to this date, using the precessions, secular variations, and proper motions given in the Catalogue, except in the case of *β Columbæ*, for which I have used the proper motion given in the Melbourne General Catalogue for 1870 instead of that given in the Cape Catalogue, which is taken from the B.A.C. It has been assumed that Henderson has corrected his places for the proper motion corresponding to the fraction of the year; but this is not quite certain; he does not give either the mean date of observation or the adopted proper motion. However, as his observations only extend from May 1832 to May 1833, the correction is insignificant, except in the case of a few stars having large proper motions. I have used 67 stars which are common to the Catalogues, situated between N.P.D. 115° and the South Pole; and proceeding in the same manner as for the northern stars, I get the following Table:—

N.P.D. °	″	Number of Stars.	H−C. Computed. ″	H−C. Curve. ″
117	34	4	+0·74	...
123	47	5	+1·04	+0·96
128	24	5	+0·47	+0·53
132	3	9	+0·24	+0·23
137	51	7	−0·40	−0·21
143	2	3	−0·16	−0·10
147	50	9	+0·19	0·00
152	51	6	−0·37	−0·12
157	40	4	+0·13	−0·13
167	7	3	−0·21	−0·15
172	52	5	−0·28	−0·24
178	26	7	−0·21	−0·22

The difference of the N.P.D.'s of these two Catalogues, as given above, are much more irregular than those of the Cape and Greenwich Catalogues; but this is no doubt owing to the inaccurate values of the proper motions of the southern stars which have been used. Mr. Stone considers that these proper motions may be 0″·02 or 0″·03 in error, which of course in 27 years would amount to a perceptible quantity. However, I have no doubt that the curve represents very fairly the systematic differences between the Catalogues as given by these observations.

As it might be considered that there were not sufficient materials for a trustworthy direct comparison between Henderson and the Cape Catalogue, I have also compared them indirectly by means of Johnson's St. Helena Catalogue. There are 202 stars available for the comparison of Johnson with the Cape, and Dr. Auwers has given the reduction of Johnson to Henderson derived from 149 stars. Johnson has not corrected his places for proper motion, except in a few instances given in the Notes, and has not given the mean dates of observation except in these cases. But as the observations extend from Nov. 1829 to April 1833 and are reduced to 1830·0, it has been assumed that 1832·0 may, in the mean, be taken as the mean date of observation, and 2 years' proper motion has been applied to every star, and then the place in the Cape Catalogue has been brought back to 1830·0. The result of the comparison is—

N.P.D.	Number of Stars.	J−C.	
		Computed.	Curve.
° ′		″	″
122 17	34	+0·63	+0·52
127 27	26	+0·68	+0·77
132 19	26	+1·00	+0·69
137 31	23	+0·01	+0·42
142 51	14	+0·74	+0·51
147 32	19	+0·65	+0·50
152 27	15	−0·32	+0·19
156 51	20	+0·55	+0·38
162 37	5	+0·67	+0·52
168 10	7	+0·26	+0·33
172 55	5	+0·24	+0·31
177 55	8	+0·47	+0·39

By applying Dr. Auwers' reduction of Johnson to Henderson and taking the mean of the direct and indirect comparison of Henderson and the Cape Catalogue, the final Table of reduction becomes—

N.P.D.	H−C.		
°	Direct.	Indirect.	Mean.
	"	"	"
120	+1·10	+1·37	+1·24
122	+1·05	+1·40	+1·23
124	+0·90	+1·43	+1·17
128	+0·60	+1·31	+0·96
132	+0·20	+0·85	+0·53
136	−0·09	+0·30	+0·11
140	−0·25	−0·05	−0·15
144	0·00	−0·09	−0·05
148	−0·01	−0·34	−0·18
152	−0·10	−0·78	−0·44
156	−0·16	−0·76	−0·46
160	−0·10	−0·54	−0·32
164	−0·08	−0·63	−0·36
168	−0·20	−0·73	−0·47
172	−0·23	−0·63	−0·43
176	−0·25	−0·35	−0·30
180	−0·23	+0·07	−0·08

Greenwich,
1878, *December* 9.

Professor Safford has kindly pointed out to me an *Erratum* in the proper motion in R.A. of 35 *Ceti*, as given in *Monthly Notices*, vol. xxxviii., p. 518. The corrected quantities are:

	s
Mean Annual Motion	+3·0661
Proper Motion in R.A.	−0·0142.

Greenwich,
1878, *December* 9.

Total Eclipse of the Sun, July 29, 1878.

By W. H. Pickering, Esq.

The eclipse was observed by me at Cherry Creek, two and a quarter miles south-west of Denver, Colorado. The instruments used were two Arago polariscopes (the same used by my brother, Professor E. C. Pickering, in 1869 and 1870) and a polarimeter lent me by Mr. Ranyard. The smaller polariscope consisted of a double-image prism and a selenite plate. It was twelve inches in length, one inch in diameter, and 4° 45′ field, giving red and

green colours. The larger polariscope was composed of a double-image prism and a quartz plate rotating to the left, giving blue and yellow. It was nineteen inches long, the quartz was one inch in diameter, and gave a field of 3°. The polarimeter consisted of two quartz wedges giving Savart's bands and a Nicol's prism, and these were neutralised by four crown-glass plates turning on a horizontal axis. The tube was nine inches in length and $\frac{3}{4}$ inch in diameter, giving a field of 4° 40′. The object of taking two Arago polariscopes was that the larger one, in 1869 in the hands of my brother, and in 1870 with Mr. W. O. Ross, had shown the corona wholly unpolarised, while the smaller instrument, at the same eclipse in 1870 with my brother, had shown the corona polarised radially. By taking both instruments and using first one and then the other, I hoped to obtain a clue to these contradictory results. Prior to the eclipse, in order to practise myself in observing radially polarised light, I had imitated this condition by laying a glass funnel on a looking glass, and reflecting the light of the sky through it. The light was thus strongly polarised—tangentially, however, instead of radially; but the effect was the same, only with the colours reversed. The artificial corona was thus seen surrounded by the blue background of the sky, which gave the usual polarisation.

The instant totality came on I observed the corona through the selenite polariscope, and found it radially polarised, but the colours were rather faint. They were the same above and below and complementary on either hand. I next took up the larger polariscope, and obtained a similar result. The right-hand image with vertically polarised light is blue when a mark upon the tube of the instrument is held uppermost. In this case, however, the right-hand image of the corona was green above and below and red on either hand, no other colours being visible. I next swept the instrument around the sky about 40° above the horizon. In determining the plane of polarisation I revolved the tube about its own axis until the image to the right of the mark attained its characteristic blue, and the other the characteristic yellow tint; the plane of polarisation then passed through the line separating the two images. The Sun at this time was nearly due west and about 40° above the horizon. When the instrument was pointed north and at the same altitude as the Sun the polarisation was nearly horizontal, and seemed to lie in a plane passing through the Sun. When pointed to the south I obtained the same result. I was surprised not to find it vertical, and repeated both observations, but with the same result. On pointing the instrument east I found the polarisation vertical, but could distinguish none whatever at the zenith. I looked very carefully at this point. I next took up the polarimeter and observed the sky about 10° to the right of the Sun and also 10° below it. In the hurry of the moment I held the instrument vertically, instead of first bringing the bands to a maximum and

then neutralising them with the glass plates. At 10° to the right I found the lines disappear when the plates were inclined 8°. At 10° below the lines were faintest at 2°.

I noticed that the night hawks came out shortly before totality and disappeared soon after. Just as totality was over I observed a number of shadowy bands on the ground. They were about a yard broad and two yards apart, and had a slightly wavering motion. They were very conspicuous and lasted nearly a minute. These estimates were made from memory immediately after the bands had disappeared.

In order to account for the colours as seen through the quartz polariscope, I have investigated the subject further by means of my funnel and mirror. I have found that when the light is only slightly polarised the blue and yellow, although in broad sectors, are not so conspicuous as the red and green, and that the green in the left-hand image, although not distributed vertically, is more nearly so than it is horizontally. My observation, if interpreted rigorously, would indicate a polarisation something between radial and tangential; but inasmuch as the colours were very faint, a slight variation on one side or the other of the vertical would hardly have been noticed, and I think it is safe to regard the result as denoting slight radial polarisation.

It is rather difficult to interpret my observations on sky polarisation. If we could discard the north and south observations, they would indicate radial polarisation from the zenith as a centre. On the other hand, accepting these and discarding the zenith observation, we have radial polarisation from the Sun. Finally, accepting these three as correct and disregarding the east observation, we have the sky horizontally polarised. The best interpretation which occurs to me is to suppose the sky not polarised merely from the zenith as a centre, nor merely from the Sun, but from both. In this case the polarisation in the east would be marked, which was true; in the zenith and in the neighbourhood of the Sun it would be very faint (the latter is shown by my polarimeter observations); and in the north and south the plane would lie neither in the direction of the zenith nor of the Sun, but somewhere between the two. In regard to my polarimeter observations: taking the one made 10° to the right of the Sun, it is evident that there was no perceptible horizontal polarisation or the lines would not have disappeared; and the vertical polarisation, if any, must have been very slight, as indicated by the number of degrees. The second observation, made 10° below the Sun, indicates no vertical, and perhaps very slight horizontal, polarisation.

Note by the Astronomer Royal on a Determination of the Mass of Mars.

Professor Asaph Hall, in his admirable paper on the satellites of *Mars*, after announcing his first inference in regard to the value of the mass of *Mars*, $\frac{1}{3107713}$, has cited the values $\frac{1}{1846082}$, $\frac{1}{2546320}$, $\frac{1}{2680337}$, $\frac{1}{3200900}$, $\frac{1}{2994790}$, $\frac{1}{2812526}$, successively adopted by Laplace, Delambre, Burckhardt, Hansen and Olufsen, and Le Verrier (Le Verrier has used the two last stated). I have not been able to trace the investigations on which these numbers are founded.

But I may perhaps call attention to an investigation made by myself, fifty-one years ago, published in the *Philosophical Transactions* for the year 1828. It is one of the results of a discussion of $10\frac{1}{2}$ years' Greenwich observations of the Sun, from 1816, July 30, to 1826, December 30. The number of observations (from which all observations made under unfavourable circumstances have been eliminated) is 1,212. The result for the mass of *Mars* is $\frac{1}{3734602}$.

The Memoir in the *Philosophical Transactions*, 1828, to which I have alluded above, has not, apparently, met the eye of our transatlantic friends. As illustrating the progress of science in defining the place of the Sun and the masses of planets and Moon, it may perhaps be worthy of a short abstract at the present time.

In the year 1827 the attention of the Board of Longitude was called (in the first instance by Mr., afterwards Sir James, South) to the state of the Solar Tables used in the *Nautical Almanac*; and I, as a member of that Board, undertook the task of comparing the tables with observations; and a mass of more than 1,200 transits of the Sun, with stars, was placed in my hands by the then Astronomer Royal, Mr. Pond. For the method of discussing these I refer to the *Philosophical Transactions*, 1828. The principal results were the following:—

The mass of *Mars* adopted by Delambre was reduced in the proportion of 9 : 5 nearly. (It is to be remarked that the perturbations produced by *Mars* are small.)

The mass of *Venus* was somewhat reduced, and the value thus obtained agrees sensibly with that now adopted.

The mass of the Moon was reduced from $\frac{1}{60}$ nearly to $\frac{1}{80}$ nearly, a value now universally adopted. I do not remember whether any preceding investigation had led to a similar result.

Corrections were found for the excentricity of the Earth's orbit and for the place of perihelion.

A correction was found for the epoch of Sun's longitude; and this correction led to a result of some importance. The comparison of this with similar corrections found by Burckhardt for other epochs convinced me that there was an unrecognised inequality in the Sun's longitude, and finally led to the discovery of the equation of long period in the motions of the Earth and *Venus*.

To these results applying to the movable bodies of the solar system, I may add one which affects the honour of the Royal Observatory as maintained by my predecessor, Mr. Pond, that the whole train of the investigations testifies strongly to the excellence of the transit observations made sixty years ago, which appear to have been as good as those of the present day.

1878, *November* 16.

On the Conjunction of Mars and Saturn, 1879, *June* 30.

By Sir G. B. Airy, Astronomer Royal.

On 1879, June 30, will occur a conjunction of *Mars* and *Saturn* of unusual interest. Adopting the elements of the *Nautical Almanac*, Mr. Dunkin, at my request, has computed the geocentric places of the centres of the two planets for every hour—and through the greater part of the time for every quarter-hour—from June 30, 2^h to 20^h. The results for every quarter-hour, from 6^h to 9^h Greenwich Mean Solar Time, are laid down in a chart, which I have the pleasure of submitting to the Society. The small movements of *Saturn* are there applied, in opposite directions, to the true movements of *Mars*; so that, with a fixed representation of *Saturn*, the relative position of *Mars* is correctly given for each of those times. The magnitudes of the two planets and the position of *Saturn's* ring are accurately represented, the lower line of the chart being parallel to the Equator.

It will be seen that the nearest approach of the two planets occurs at $7^h\ 30^m$ nearly, and that the distance between their centres is then $1'\ 12''$. These numbers will be affected by the possible errors of the planetary tables.

In the table which accompanies the chart Mr. Dunkin has given the local times at Vienna, the Cape of Good Hope, Madras, Sydney, San Francisco, and Cordoba (Argentine Confederation), corresponding to the Greenwich hours above mentioned. (These stations are selected as giving a chain of longitudes proceeding easterly round the Earth, with some range of latitude.) The local times of planet-rise and sunrise are also given, as limiting the times of observation. It is mortifying to us to remark that the planets cannot be seen at their closest proximity in Europe

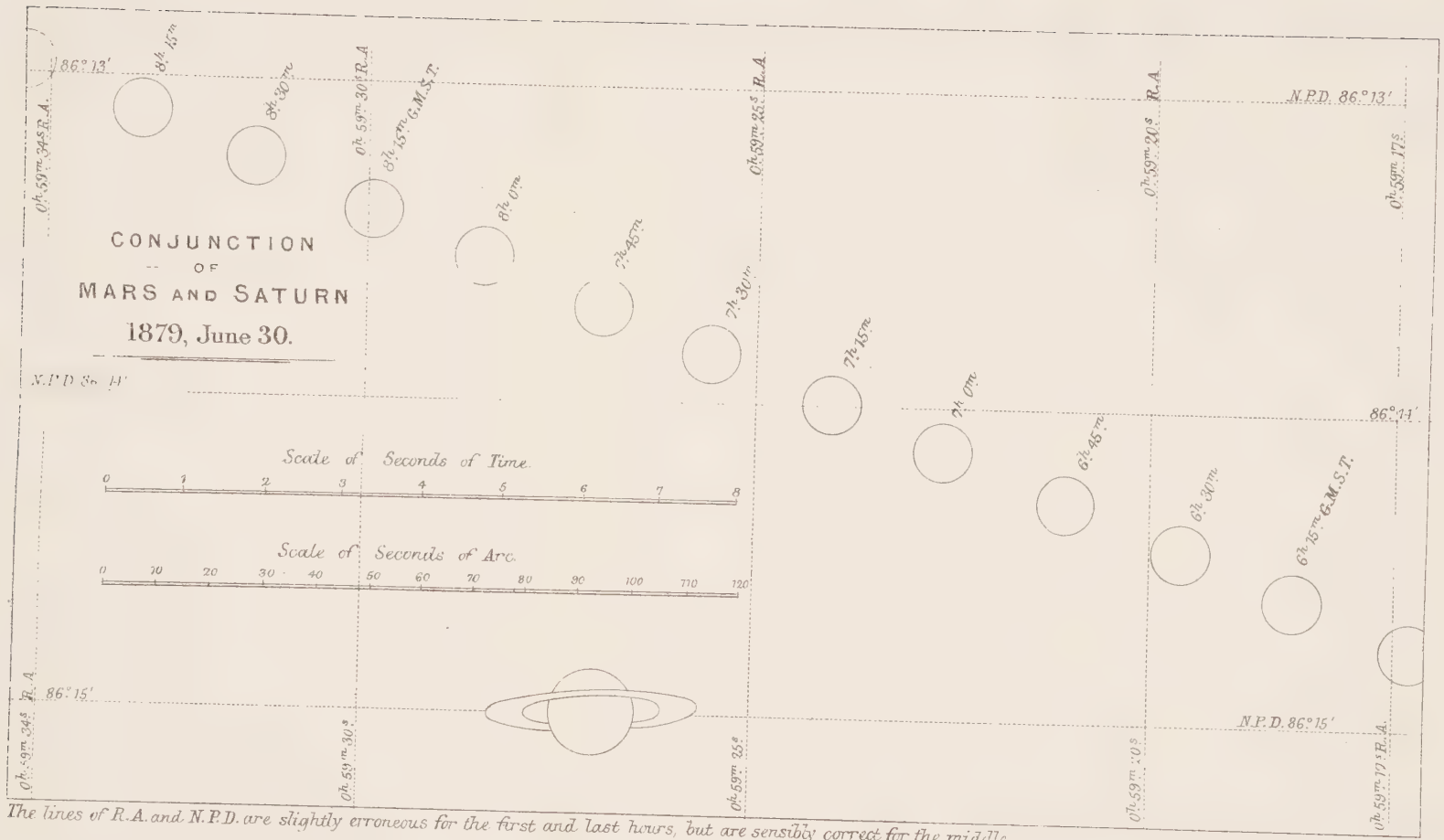
or Africa. For us the smallest interval will be about 9′. This, however, is less than the smallest interval at the conjunction of 1877, November 3. In India and Australia the conjunction will be seen extremely well. But no part of the near approach can be seen in America.

Different observers will make use of this phenomenon for different purposes. The instrumental measurement of the relative positions of the planets will give a test of Leverrier's Tables in parts of their orbits where it could not otherwise be obtained. I look with much interest to observations of the colours of the two planets. At the conjunction of 1877, November 3, the distance between the planets rendered it impossible to see them in the same field with a high power on the telescope, and therefore impossible to receive from them any large pencil of light. I did myself observe them with a good surveyor's hand-telescope, and was much struck with the result as to their colours. While *Mars* had his usual fiery yellow-red colour, *Saturn* was of deep sap-green. How much of this colour was due to contrast with the predominant blaze of *Mars* I cannot judge. In the approaching conjunction *Mars* will be much smaller, and the colours can perhaps be compared more justly.

I am permitted by Mr. Marth to state, as the result of his calculations of the places of the satellites of *Saturn*, that *Mars* will apparently pass through some of their orbits, and in particular will probably be within 30″ of *Japetus*. The nearest approach, however, will occur between 2^h and 3^h Greenwich Mean Solar Time, and probably can only be seen in some of the islands of the Pacific, as the Sandwich Islands or the Society Islands.

Mr. Marth also remarks that the position angle of the greatest defect of illumination of the disk of *Mars* on June 30, at 8^h, is 247°·08, and the greatest defect of illumination is 0·159 × diameter of the planet, or 1″·4, assuming the diameter as 8″·8.

Royal Observatory, Greenwich,
1878, *December* 7.



The lines of R.A. and N.P.D. are slightly erroneous for the first and last hours, but are sensibly correct for the middle.

Spottiswoode & Co. Lith. London.

Distances between the Centres of Mars and Saturn near their Conjunction on 1879, *June* 30, *for Local Times at Greenwich and seven other Observatories.*

Local Mean Solar Time at																			Distance of Centres of Planets.
Greenwich.			Vienna.			Cape of Good Hope.			Madras.			Sydney.			San Francisco.			Washington.			Cordoba.			
June			June			June			June			June			June			June			June			
d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	d	h	m	′ ″
30	2	0	30	3	6	30	3	14	30	7	21	30	12	5	29	19	4	29	20	52	29	21	44	8 25
	3	0		4	6		4	14		8	21		13	5		20	4		21	52		22	44	6 53
	4	0		5	6		5	14		9	21		14	5		21	4		22	52	29	23	44	5 22
	5	0		6	6		6	14		10	21		15	5		22	4	29	23	52	30	0	44	3 51
	5	30		6	36		6	44		10	51		15	35		22	34	30	0	22		1	14	3 10
	6	0		7	6		7	14		11	21		16	5		23	4		0	52		1	44	2 30
	6	30		7	36		7	44		11	51		16	35		23	34		1	22		2	14	1 54
	6	45		7	51		7	59		12	6		16	50	29	23	49		1	37		2	29	1 37
	7	0		8	6		8	14		12	21		17	5	30	0	4		1	52		2	44	1 24
	7	15		8	21		8	29		12	36		17	20		0	19		2	7		2	59	1 15
	7	30		8	36		8	44		12	51		17	35		0	34		2	22		3	14	1 12
	7	45		8	51		8	59		13	6		17	50		0	49		2	37		3	29	1 17
	8	0		9	6		9	14		13	21		18	5		1	4		2	52		3	44	1 27
	8	15		9	21		9	29		13	36		18	20		1	19		3	7		3	59	1 43

Distances between the centres of Mars and Saturn near their conjunction on 1879, *June* 30, *for Local Times at Greenwich and seven other Observatories*—continued.

Local Mean Solar Time at									Distance of Centres of Planets.
Greenwich.	Vienna.	Cape of Good Hope.	Madras.	Sydney.	San Francisco.	Washington.	Cordoba.		
June d h m	June d h m	June d h m	June d h m	June d h m	June d h m	June d h m	June d h m		′ ″
30 8 30	30 9 36	30 9 44	30 13 51	30 18 35	30 1 34	30 3 22	30 4 14		2 3
8 45	9 51	9 59	14 6	18 50	1 49	3 37	4 29		2 24
9 0	10 6	10 14	14 21	19 5	2 4	3 52	4 44		2 45
9 15	10 21	10 29	14 36	19 20	2 19	4 7	4 59		3 7
9 30	10 36	10 44	14 51	19 35	2 34	4 22	5 14		3 30
9 45	10 51	10 59	15 6	19 50	2 49	4 37	5 29		3 53
10 0	11 6	11 14	15 21	20 5	3 4	4 52	5 44		4 16
10 30	11 36	11 44	15 51	20 35	3 34	5 22	6 14		4 59
11 0	12 6	12 14	16 21	21 5	4 4	5 52	6 44		5 43
11 30	12 36	12 44	16 51	21 35	4 34	6 22	7 14		6 27
12 0	13 6	13 14	17 21	22 5	5 4	6 52	7 44		7 11
13 0	14 6	14 14	18 21	30 23 5	6 4	7 52	8 44		8 43
				July					
14 0	15 6	15 14	19 21	1 0 5	7 4	8 52	9 44		10 18

15 0	16 6	16 14	20 21	1 5	8 4	9 52	10 44	11 52
16 0	17 6	17 14	21 21	2 5	9 4	10 52	11 44	13 27
17 0	18 6	18 14	22 21	3 5	10 4	11 52	12 44	15 2
18 0	19 6	19 14	30 23 21	4 5	11 4	12 52	13 44	16 37
			July					
19 0	20 6	20 14	1 0 21	5 5	12 4	13 52	14 44	18 12
30 20 0	30 21 6	30 21 14	1 1 21	1 6 5	30 13 4	30 14 52	30 15 44	19 48

Local Mean Solar Time of Planet-rise.

30 12 5	30 12 7	30 12 34	30 12 23	30 12 32	30 12 13	30 12 13	30 12 34	

Local Mean Solar Time of Sunrise.

30 15 53	30 16 9	30 19 10	30 17 40	30 19 10	30 16 45	30 16 42	30 19 4	

On the Visibility of Stars in the Pleiades to the Naked Eye.

By Professor Winnecke.

I had lately to look over the *Historia Cœlestis Lucii Barretti*,* Augustæ Vindelicorum, MDCLXVI. There I found, on p. lxxxv., some observations of the *Pleiades* made 1579, December 24, by Moestlin† (afterwards preceptor of Kepler at Tübingen, then Diaconus at Baknang, in Wurtemberg), which are very interesting, forming indeed the *first* special catalogue of this important cluster. In the excerpta of the manuscripts of Moestlin, made by Shickard and printed by Curtz in his *Historia Cœl.*, we find the following statement.

"1579, December 24, *Plejades* ita reperit. Declinationem primæ 22° 32′, secundæ 22° 52′, tertiæ 22° 48′, quartæ 22° 23′, quintæ 22° 36′, sextæ 22° 37′, septimæ 22° 38′, octavæ 22° 42′, nonæ 22° 57′, decimæ 23° 12′, undecimæ 22° 4′. Ascensiones rectas illarum numeravit à prima, scilicet secundæ 7′, tertiæ 16′, quartæ 22′, quintæ 39′, sextæ 1° 0′, sept. 59′, octavæ 2′, nonæ 15′, decimæ 359° 58′, ultimæ 42′."

There is also given a coarse woodcut representing the stars, which it is not worth while to reproduce because it contains several obvious mistakes, probably due to the circumstance, that in the woodcut only 10 stars are given, whilst 11 were observed by Moestlin. It is possible that Moestlin has *seen* so many as 14 stars with the naked eye; for Kepler, in his *Dissertatio cum Nuncio Sidereo* (*Opera Collecta*, vol. ii. p. 500) relates, "Moestlinus majusculas in Plejadibus ordinaria numerat, nisi fallor, 14."

Employing Moestlin's *measures*, we get the following map of the stars seen by him in the *Pleiades*, 1579, December 24 (see p. 148).

There is no difficulty in identifying these stars as the following in Bessel's Catalogue :—

	m		m
1 = Electra	4·5	7 = Plejone	5·6
2 = Taygeta	5	8 = Celæno	5·6
3 = Maja	5	9 = 21 *k* Plej.	7·8
4 = Merope	5	9 = 22 *l* „	7·8
5 = Alcyone	3·4	10 = 18 *m* Plej.	7
6 = Atlas	4·5	11 = Anon Bess. 25	8·9
		11 = Anon Bess. 26	9

Reducing Bessel's places of these stars to 1580·0 without proper motion, which, for our purpose, may be considered the same for all stars of the cluster, we get :—

* Anagram of Alberti Curtii, the Latin name of the Jesuit, Albert Curtz, 1671.

† Michael Moestlin, born 1550, September 30, at Göppingen; died 1631, December 20, at Tübingen.

		R.A. 1580·0.		Decl. 1580·0.		M—B.	Error.
		°	′	°	′	′	′
1	Electra	50	2·1	+22	42·6	+10·6	+0·8
2	Taygeta	50	6·2	23	4·0	12·0	+2·2
3	Maja	50	15·6	22	58·3	10·3	+0·5
4	Merope	50	24·0	22	33·4	10·4	+0·6
5	Alcyone	50	40·8	22	43·4	7·4	−2·4
6	Atlas	51	5·4	22	41·1	4·1	−5·7
7	Plejone	51	5·5	22	46·1	8·1	−1·7
8	Celæno	50	0·6	22	53·1	11·1	+1·3
9	$\frac{21k+22l}{2}$	50	17·5	23	8·8	11·8	+2·0
10	$18m$	50	4·7	23	26·3	14·3	+4·5
11	$\frac{\text{An. } 25+\text{An. } 26}{2}$	50	44·7	22	11·8	7·8	−2·0

The column M−B contains the difference between the declinations of Moestlin and Bessel; the last column the outstanding error after applying a constant correction of +9′·8 to the former.

Putting the R.A. of *Electra* = 0, we get the difference in R.A.

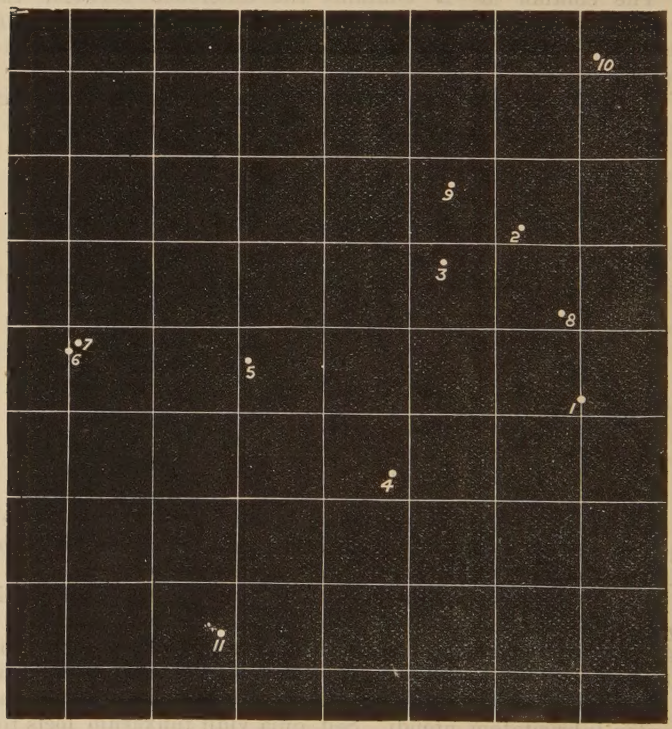
	Bessel.	Moestlin.	M—B.
	′	′	′
(2)−(1)	+ 4·1	+ 7	+2·9
(3)−(1)	13·5	16	+2·5
(4)−(1)	21·9	22	+0·1
(5)−(1)	38·7	39	+0·3
(6)−(1)	63·3	60	−3·3
(7)−(1)	+63·4	59	−4·4
(8)−(1)	− 1·5	2	+3·5
(9)−(1)	+15·4	+15	−0·4
(10)−(1)	2·6	− 2	−4·6
(11)−(1)	+42·6	+42	−0·6

The sum of the errors (each error as positive) is for Decl. 23′·7, for R.A. 22′·6. The probable error of a difference of R.A. or Decl. is therefore nearly the same and about ±2′.

The method used by Moestlin in getting his observations is not stated, but it is probable that the measures were taken by a Radius astronomicus just finished by his own hands. In the excerpta occurs the following statement:—"Medio Decembris (1579) fabrefecit radium observatorium, quo prius uti noluerat propter lubricitatem utendi. Sed quia vidit infixarum locis tot errores, fabrefecit. Nil enim prosunt planetarum observationes, si fixarum situs falsi sunt. Ea fuit causa, quæ decepit canonis planetici autores etc.; has prius reformare debuissent. Regulam

sumpsit 14 pedes longam, dividens in 14 particulas, et aptavit diversa transversaria partium 10, 16, 20, 32, 40, 60, 80, 100, 160, 200, 300, 400, 600, 800, iis pinnacidia junxit ad extremitates. Regulæ ipsi prope oculum infixit cuspidem certius collinandi gratia."

The little catalogue, given above, appears to be of some importance. Moestlin made it at a time *when no telescope had given evidence* that there are a great many stars in the *Pleiades*. The accuracy of their relative places is truly astonishing in so difficult an object. It is most interesting to compare the chart of the *Pleiades* founded on Moestlin's measures with that given by the Astronomer Royal (*Monthly Notices*, vol. xxiii. p. 175). Ten of the stars seen by both observers are identical without a doubt, and perhaps the 11th star is also the same as that seen at Greenwich, with a large error in Moestlin's measures. The 12th star seen at Greenwich is possibly also shown in the coarse woodcut mentioned above.



Strassburg,
1878, *December* 3.